CERN-PH-TH/2004-143
CU-TP-1123
MAD-TH-04-07
hep-th/0411217

# Decoupling in an expanding universe: backreaction barely constrains short distance effects in the CMB.

**Brian R. Greene** [1,2], **Koenraad Schalm** [1], **Gary Shiu** [3], **and Jan Pieter van der Schaar** [4]

[1] Institute for Strings, Cosmology and Astroparticle Physics &
Department of Physics, Columbia University, New York, NY 10027
[2] Department of Mathematics, Columbia University, New York, NY 10027
[3] Department of Physics, University of Wisconsin, Madison, WI 53706
[4] Department of Physics, CERN, Theory Division, 1211 Geneva 23

E-mail: `greene@phys.columbia.edu`, `kschalm@phys.columbia.edu`, `shiu@physics.wisc.edu`, `jan.pieter.van.der.schaar@cern.ch`.

**Abstract.** We clarify the status of transplanckian effects on the cosmic microwave background (CMB) anisotropy. We do so using the boundary effective action formalism of `hep-th/0401164` which accounts quantitatively for the cosmological vacuum ambiguity. In this formalism we can clearly 1) delineate the validity of cosmological effective actions in an expanding universe. The corollary of the initial state ambiguity is the existence of an earliest time. The inability of an effective action to describe physics before this time demands that one sets initial conditions on the earliest time hypersurface. A calculation then shows that CMB anisotropy measurements are generically sensitive to high energy corrections to the initial conditions. 2) We compute the one-loop contribution to the stress-tensor due to high-energy physics corrections to an arbitrary cosmological initial state. We find that phenomenological bounds on the backreaction do not lead to strong constraints on the coefficient of the leading boundary irrelevant operator. Rather, we find that the power spectrum itself is the quantity most sensitive to initial state corrections. 3) The computation of the one-loop backreaction confirms arguments that irrelevant corrections to the Bunch-Davies initial state yield non-adiabatic vacua characterized by an energy excess at some earlier time. However, this excess only dominates over the classical background at times before the 'earliest time' at which the effective action is valid. We conclude that the cosmological effective action with boundaries is a fully self-consistent and quantitative approach to transplanckian corrections to the CMB.

Submitted to: *JCAP*

PACS numbers: 04.62.+v, 11.10.Gh,98.80.Cq,98.80.Qc

## 1. Introduction

During the past few years, the prospect of probing ultra high energy physics (on the order of the string/Planck scale) through precision cosmological observations has received much attention. While more traditional accelerator based approaches have the capacity to probe string physics only if the string scale proves to be in the TeV range, the enormous stretching of distance scales in inflationary scenarios may well turn the cosmos into a *Planck-scale* microscope: In all but the most conservative of inflationary models, early-universe Planck scale physics is stretched by cosmological expansion to scales relevant to the production of CMB temperature fluctuations [1, 2]. This suggests the tantalizing possibility that measurements of CMB anisotropies may be a window onto Planck scale physics.

The essential quantitative question facing this program is: How large are the contributions coming from string/Planck scale physics, and are they large enough to be observed? Many authors, using a variety of approaches, have reached a range of conclusions. The most optimistic estimates have calculated transplanckian contributions to the CMB power spectrum ranging from $O(1)$ to $O(H/M_{\text{string}})$ [3, 4, 5, 6, 7] with $H$ the Hubble scale at the end of inflation, while more pessimistic estimates [8] suggest that the maximum contributions are order $O((H/M_{\text{string}})^2)$ (see further [9]). With a favorable value of $H/M_{\text{string}} \sim 10^{-2}$, the former contribution would be on the edge of visibility [10], while the latter contributions (without anomalously large coefficients) would fall below the limit of cosmic variance. Achieving consensus on the likely size of transplanckian corrections to the CMB power spectrum is thus an issue of great importance.

In this paper, we focus on transplanckian effects on the CMB coming from the well known and much studied ambiguity in choosing a vacuum state in a nontrivial cosmological space-time. By way of brief history, in [3] a combination of such a modified vacuum state together with string-motivated modifications to the dynamical evolution equations were studied, with the conclusion that order $H/M_{string}$ corrections are generic. In [6] and the third article of [3], it was emphasized that a modified vacuum state together with *standard* evolution equations would also give rise to generic $H/M_{string}$ corrections, thus clarifying and simplifying the origin of transplanckian contributions of this magnitude. However, the papers [8], employing effective field theory techniques, argued that the standard vacuum state (Bunch-Davies boundary conditions) would yield the far smaller $(H/M_{\text{string}})^2$ contributions, and that, moreover, any other choice for the boundary conditions would fall prey to uncontrollable backreaction in the absence of fine tuning. Inspired in part by [7], in [11] a quantitative formalism for studying the ambiguity in cosmological initial conditions in the context of boundary effective field theory was put forward. It was argued that the integrating out of high energy modes *generically* modifies the boundary conditions of the resulting low energy effective field theory. Importantly, the size of the corrections was found to be of order $H/M_{string}$, agreeing with the early papers on high energy effects on the vacuum choice, but with calculations carried out in a controlled approximation. Moreover, the boundary effective action provided new counterterms that would render the gravitational backreaction under control for choices other than Bunch-Davies boundary conditions. Subsequently, an order of magnitude estimate of the now manifestly finite backreaction [12] argued that the observed slow-roll period of inflation significantly bounds the size of potential transplanckian signatures in the CMB anisotropy.

In this paper, armed with the quantitative boundary effective field theory framework, we extend the work of [11] and [12] by carefully calculating the gravitational backreaction. Seeking fully unambigous backreaction constraints — backreaction contributions that are not subject to any renormalization ambiguities and hence are fully interpretable in the effective field theory framework — we find robust conclusions on the size of transplanckian CMB corrections. In particular, the existence of new counterterms has the potential to subdue the order of magnitude estimate of the one-loop backreaction to the point that there are no bounds of consequence on potential corrections to the observed power spectrum for $H/M > 10^{-4}$. Without additional information from a proposed UV-completion of our effective field theory description — one that would allow a more refined estimate of the gravitational backreaction — we conclude that backreaction constraints are under control for parameter ranges that can, in principle, yield observable consequences for the cosmic microwave background radiation. Barring a cosmological observation more sensitive to high-energy corrections to the initial state, the theoretical window of opportunity to observe short distance physics in the CMB is thus open.

### *1.1. Summary of Results*

In section 2 we review the results of [11] that provide a new formalism to address the initial state ambiguity in cosmological space-times in a coherent Lagrangian effective field theory description. The initial state can be encoded in a (space-like) boundary action at an a priori arbitrary initial time $t_0$. From the standard Hamiltonian perspective, this corresponds to scenarios where the Bogoliubov coefficients parameterizing the initial state ambiguity are allowed to vary with co-moving momentum $k$. The distinct advantage of the Lagrangian effective field theory approach is that it is the natural framework to consider effects of high energy physics (see e.g. [13]). It allows one to neglect momentum modes beyond the high energy cut-off in a manifestly consistent way and it provides a *quantitative* understanding of effects due to UV physics. Specifically, it recasts the momentum-dependence of the initial state into a well-defined expansion in irrelevant operators parameterizing unknown high-energy physics.

The leading effect of high-energy physics arises from these irrelevant corrections to the boundary action rather than from higher derivative corrections in the bulk (which were analyzed in [4, 8]). The leading irrelevant operator on the three-dimensional boundary is dimension four. Corrections to the inflationary power spectrum therefore behave parametrically as $H/M$, which can conceivably be as high as 1%. This renders them potentially observable in future CMB measurements.

*An earliest time* As we will discuss in 2.1, an immediate consequence of the effective field theory framework in a cosmological context — of demanding that all the momentum modes under consideration are *always* smaller than the physical cutoff scale — is the existence of an 'earliest time' before which the effective field theory breaks down. This earliest time depends on the momentum scale of interest and roughly corresponds to $\ln(M/H)$ e-folds before horizon exit of the smallest observable length scale in the CMB. It is the natural location to set the initial conditions.†

By doing so, we find that the current measurements of the power spectrum of CMB fluctuations already provide a strong bound on the *coefficient* $\beta$ of the leading irrelevant operator in the boundary effective action if $H/M > 10^{-4}$. This is directly related to the fact that the initial state deduced from the boundary effective field theory will generically break de Sitter scale invariance, (i.e. the Bogoliubov coefficients depend on the co-moving momentum scale in Hamiltonian language). Roughly put, the effect of the leading irrelevant boundary operator adds a linear component $\delta P = \beta k/a_0 M$ to the power spectrum, where $a_0$ is the scale factor at the 'earliest time'. The change to the power spectrum is thus parametrically controlled by $k/aM \sim H/M$, but at the high end of the spectrum it can be significantly larger than that (figure 1 in section 2.1). This linear growth is not present in the observed CMB spectrum; it is nearly scale invariant over the full observed range. This rules out a coefficient $\beta$ much larger than 0.1.

We see that an earliest time has as primary consequence that inflation's usual classical independence of initial conditions is modified as the quantum mechanical boundary conditions affect late-time physics. In particular, if the modes responsible for the CMB perturbations that we can measure today exited the horizon a sufficiently small number of e-foldings after the earliest time hypersurface, then sensitivity to initial conditions can persist. Nevertheless, the cosmological dynamics that result from the classical background motion of the scalar field are still expected to be independent of the initial conditions [14].

*Backreaction constraints* Naturally, all other — measured — cosmological quantities will also be affected by the irrelevant boundary operators and observability therefore

† In practice this natural proposition raises a question. For optimistic inflationary scenarios with $M/H \sim 10^2$, the lowest modes observable in the CMB — four orders of magnitude below the smallest observable mode — have already exited the horizon $\ln(10^2)$ e-folds earlier. Usually horizon exit is interpreted as modes ceasing to be dynamical and being frozen out. If this interpretation is truly correct, how can these modes account for low multipole CMB fluctuations? At face value, 1) the imposition of boundary conditions at a fixed 'earliest' time, 2) the non-uniform times *a*) when each individual mode is of the order of the cut-off and *b*) when it exits the horizon, and 3) the wish to describe the full range of observed modes in the CMB at once, are in tension with each other. For the power spectrum we can circumvent this conflict as a linear analysis is sufficient. Modes do not interact, and one can set the initial conditions for each mode at different times. For an interacting field theory it is open question , however, how to resolve the tension between 1), 2) and 3).

hinges on whether other phenomenological constraints are mild enough to allow a 1% change to the power spectrum. In particular, [12] argued that an order of magnitude estimate of the gravitational backreaction yields constraints on transplanckian calculations that are quite significant.‡ We compute here the gravitational backreaction in detail. The resulting perturbative bound on the coefficient $\beta$ of the leading irrelevant boundary operator,

$$|\beta|^2 \ll (12\pi)^2 \left(\frac{M_p^2 H_0^2}{M_{string}^4}\right) , \tag{1}$$

plus the constraints from the observed inflationary slow-roll parameters $\epsilon_{observ}, \eta_{observ}$

$$|\beta|^2 \lesssim 2\,(6\pi)^2 \,|\epsilon_{observ}| \left(\frac{M_p^2 H_0^2}{M_{string}^4}\right) , \tag{2}$$

$$|\beta|^2 \lesssim (6\pi)^2 \,|\epsilon_{observ}|\, |\eta_{observ}| \left(\frac{M_p^2 H_0^2}{M_{string}^4}\right) , \tag{3}$$

entail relatively mild backreaction constraints. For typical but optimistic values for $H \sim 10^{14}$ GeV, the scale of new physics $M_{string} \sim 10^{16}$ GeV, and the reduced Planck mass $M_p \sim 10^{19}$ GeV, they allow a significant observational window of opportunity. The mildness results from the fact that the first unambiguous contribution to the backreaction is only at *second* order in the irrelevant correction (This had earlier been argued by Tanaka [15]. Indeed compared to the order of magnitude estimate [12] the above three equations are effectively the same with $\beta^2$ substituted for $\beta$.) The backreaction due to the first order correction, though not zero, is essentially localized on the boundary and therefore subject to the subtraction prescription utilized to renormalize the theory. The localization is a consequence of the highly oscillatory nature of the first order power-spectrum. When integrated, all contributions cancel except in a neighborhood of the boundary. In the context of effective field theory — the only context we consider in this paper — such terms are subject to renormalization via boundary counterterms and hence their contribution is ambiguous. By contrast, the second order effect which remains and dominates is the 'time-averaged' energy stored in the oscillatory behavior itself, which gives a contribution whose scale is controlled by $H$ and hence is clearly physical. In particular, this contribution grows as the square of the amplitude rather than linearly, and it is this which accounts for the appearance of $|\beta|^2$ rather than $|\beta|$ in eqs. (1)- (3) above.

The bounds on the coefficient $\beta$ due to the one-loop backreaction are in fact so mild that they are superseded by the direct sensitivity of the power spectrum. The aforementioned existence of an earliest time and its concomitant bound on $\beta \leq 0.1$ implies that backreaction poses *no* constraints if $H/M$ is large enough (figure 1 in section 2.1). The bounds on $\beta$ from backreaction are weaker than the direct 'search' upper

‡ That backreaction effects in this context could be important was also emphasized in [15] (see also [16] and the recent articles [17, 18])). Other phenomenological constraints on initial state modifications have been discussed in [19]. More formal arguments against the use of non-standard initial states can be found in [8, 20].

bound from the power-spectrum, allowing for the possibility of non-trivial implications for CMB observations.

*Energy and adiabaticity of the initial state* The calculation of irrelevant contributions to the one-loop backreaction does make clear that the presence of irrelevant corrections to the Bunch-Davies boundary action must amount to an extra vacuum energy presence in the space-time. The non-localized second order contribution to the backreaction redshifts away as any other energy density. Qualitatively this indicates that the effective action has a limited range of applicability to the far past to where the quantum vacuum energy overwhelms the classical one, as has been argued in [8]. The boundary action formalism allows a quantitative calculation of this range of applicability. One finds that the excess energy stored in the initial state only dominates *before* the earliest time where the effective action ceases to be valid. Within this formalism we are therefore intrinsically unable to answer what happens before that moment. This is not to say that the criticism that the excess energy could ruin inflation a few $e$-folds further back is without merit. This is certainly an issue, and one that would need to be addressed by fully fundamental description. But within the framework of a low energy effective action it is an issue that we need not, and in fact, cannot address: by definition one cannot answer any questions outside the effective field theory's range of validity. The whole framework is therefore self-consistent, and it crucially hinges on the existence of an earliest time. Within this range of validity backreaction never gets out of hand.

Beyond consistency, one might worry about fine-tuning: How finely do we have to tune the initial state to obtain observably large transplanckian effects in the CMB without falling afoul of a theoretical or phenomenological constraint? We will address this briefly in the conclusion, section 5. A fine-tuning requirement itself is no phenomenological constraint, however, but a theoretical prejudice. It is not related to consistency of the boundary effective action framework or phenomenological bounds on potential short distance effects in the CMB. For this reason we postpone a full accounting of fine-tuning issues to a separate article.

Since energy is stored in the initial state, this does show that a non standard choice of initial conditions necessarily corresponds to a state differing from the adiabatic ground state of the system. The irrelevant corrections to the initial state therefore parameterize deviations from adiabaticity.

The above demonstrates that the use of a boundary effective action to parameterize the cosmological vacuum ambiguity, allows us to *quantitatively and reliably* describe and compute high energy corrections to low energy and late time cosmological physics, including those to the initial conditions. The calculations confirm qualitative estimates about phenomenological constraints on irrelevant non-adiabatic corrections to the initial state, with important caveats. The backreaction is only significantly affected at second order. Hence 1) the size of phenomenologically allowed irrelevant corrections is significantly larger than dimensional analysis would suggest and 2) for $H/M > 10^{-4}$ the inflationary power spectrum of fluctuations itself is the most sensitive measurement of irrelevant corrections to the initial conditions. In particular the corrections to the power spectrum are allowed to be of order 1% without significantly disturbing the inflationary background. As a consequence backreaction poses little constraint and initial state corrections due to the irrelevant operators could be large enough to be observed. Indeed given that the backreaction bounds are so weak, the observed scale invariance of the CMB bounds $\beta \lesssim 0.1$ *directly.*

## 2. Initial states in effective field theory and initial states: a brief review

For simplicity we will consider an external scalar field in conformal gauge de Sitter. This situation is applicable for tensor-fluctuations in the CMB. We expect our results to carry over to the dominant (and measured) scalar-fluctuations without significant change beyond the known amplitude magnification.

In the formalism of [11] the cosmological vacuum choice ambiguity is captured by a boundary action at an arbitrary initial time $t_0$. A specific choice of boundary couplings corresponds to a specific choice of initial state. For $\lambda\phi^4$ theory on a semi-infinite space§

$$S_{bulk} = \int_{t_0 \le t \le \infty} d^3x dt \sqrt{g} \left( -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \right) , \tag{4}$$

with the following boundary interactions

$$S_{boundary} = \oint_{t=t_0} d^3x \sqrt{g_{\mu\nu}\partial_i x^\mu \partial_j x^\nu} \left( -\frac{\kappa_0}{2}\phi^2 \right) , \tag{5}$$

variation of the bulk and boundary action‖ yields the usual bulk equations of motion *plus* a boundary term that vanishes when the normal derivative of $\phi$ obeys

$$\partial_n \phi|_{t=t_0} = -\kappa_0\, \phi(t_0) . \tag{6}$$

This initial condition on the (classical) fields encodes the quantum state or equivalently the scalar field wave-functional at $t = t_0$. The specific value of $\kappa_0$ uniquely determines the boundary condition: for $\kappa_0 = 0$ we have Neumann boundary conditions, for $|\kappa_0| \to \infty$ we find a Dirichlet boundary condition and for finite $\kappa_0$ some mixture of the two. For constant $\kappa$ the boundary action is renormalizable. One can impose different boundary conditions for different (spatial) momentum modes by choosing a momentum dependent effective $\kappa_{eff}(k)$. Expanding around $k = 0$ following the precepts of effective actions, returns the power-counting renormalizable (relevant and marginal) couplings $\kappa_0$ and $\kappa_1(k) = \alpha|k|$ plus a set of nonrenormalizable higher derivative (irrelevant) operators $\kappa_n(k) = \beta_{n-1}|k|^n/M^{n-1}$. These represent our ignorance of (very) high energy physics beyond the physical cut-off scale $M$. In a flat Minkowski $\kappa_{eff}(k)$ is uniquely determined

§ We consider 4-dimensional Lorentzian space-times and introduce a space-like 3-dimensional boundary, allowing the boundary dynamics to have a natural interpretation in terms of how they affect the initial state. Working with effective Lagrangians we will implicitly assume that our results can be obtained by a Wick rotation from Euclidean space.

‖ The variations on the boundary are arbitrary; otherwise we would be imposing Dirichlet boundary conditions.

by Lorentz symmetry: $\kappa_{Mink}(k) = -i\omega(k)$. In cosmological settings the absence of Lorentz symmetry allows *a priori* more general initial conditions, including irrelevant corrections due to unknown high-energy physics.¶ In [11] all the leading dimension four irrelevant operators respecting the homogeneity and isotropy of FRW cosmologies were constructed and analyzed. In this letter we will focus our attention on one of them:+

$$S^{irr.op.}_{bound} = \oint_{\eta=\eta_0} d^3x\, a_0 \left( -\frac{\beta}{2M} \partial^i \phi \partial_i \phi \right) . \tag{7}$$

Here $a_0 \equiv a(\eta_0) = -1/H\eta_0$ is the de Sitter scale factor evaluated on the boundary surface at conformal time $\eta = \eta_0$. This specific irrelevant operator is also the one analyzed in [12]. It leads to the following $\mathcal{O}(1/M)$ corrections to the relevant coupling $\kappa_0$

$$\kappa_{eff}(k) = \kappa_0 + \beta \frac{k^2}{a_0^2 M} . \tag{8}$$

The dimensionless coefficient $\beta$ is in principle determined by the UV completion of the theory and is expected to be of $\mathcal{O}(1)$ if $M$ is the scale of new physics (although the possibility of fine-tuned coefficients cannot be excluded, of course). The phenomenological importance of a $\beta \sim \mathcal{O}(1)$ irrelevant correction is that it leads to corrections to the inflationary power spectrum parametrically controlled by $H/M$, that might be detectable in future CMB observations. Explicitly, the change in the power spectrum due to the irrelevant operator (7) is given by

$$\frac{P + \delta P}{P}(y_0) = \left( 1 + \frac{\pi \beta H}{4M} \left( i \bar{\mathcal{H}}^2_{3/2}(y_0) y_0^2 + \text{c.c} \right) \right) , \tag{9}$$

where $y_0 \equiv k/a_0 H$ is the physical momentum in units of the horizon-size at the time where we set the initial conditions, and $\mathcal{H}_{3/2}(y_0)$ is the Hankel function. (For details underlying this result we refer to [11].) Because these corrections will break the scale invariance of the inflationary power spectrum (see Figure 1), a coefficient of $\mathcal{O}(1)$ is in fact already directly excluded from the observed CMB spectrum.* Our aim here will be to show that the observed scale invariance is truly a direct bound on the size of $\beta$ and not superseded by phenomenological bounds on $\beta$ due to gravitational backreaction.

### *2.1. An earliest time in cosmological effective actions. The inflationary power spectrum*

Perturbative effective actions are intrinsically limited in their range of validity to scales below the physical cut-off $M$. In cosmological effective actions this means that the action

¶ Of course, the absence of Lorentz symmetry is by itself not enough to create a vacuum/initial state ambiguity. If there exists a global timelike Killing vector the vacuum is uniquely determined [21, 22].

+ The one-loop backreaction, which is the goal of this letter, is dominated by the high-$k$ modes and all leading dimension 4 operators reduce to eq. (7) in the high $k$ limit. In the language of [11] the coefficient $\beta$ equals $\beta = \beta_{\|} - \beta_c - \beta_{\perp}$.

* Scale invariance as a characteristic of the inflationary power spectrum is especially emphasized in [23].

is expected to break down both for high scales and early times where all momenta are blueshifted. A strength of the boundary effective action formalism is that the momentum expansion is manifestly controlled by two small parameters: the bulk action is controlled by $k/a(t)M$, the boundary action by $k/a_0M$. We immediately see that an FRW effective action is only valid up to the 'scale'

$$\mu_{phys}(t) \equiv \frac{\mu_{co}}{a(t)} = M \ , \tag{10}$$

and only valid up to a 'time'

$$a_0 = \mu_{co}/M \ . \tag{11}$$

where $\mu_{co}$ is the scale of interest in units co-moving momentum; i.e. the largest comoving momentum mode we can see in today's CMB. We see here the mathematical manifestation of our intuition that we can only trust low energy effective cosmological theories up to the 'Planck time'.

This 'earliest time' is the logical place to locate the boundary action to set the initial conditions. Doing so, we can refine our analysis for which values of $\beta$ and $H/M$ changes in the power spectrum are of the right order of magnitude to be potentially observable.

Clearly the maximal change in the power spectrum occurs for the largest possible value of $y_{0,max} = k_{max,observed}/a_0H$. This is simply a consequence of the fact that we are studying the effects of irrelevant operators whose size increases with $\vec{k}$. Having realized the existence of an earliest time $a_0 = k_{max}/M$, we set the initial conditions there (we *cannot* choose an earlier time with $a_0$ less than that; we could choose a larger value at a later time). Hence $y_{0,max} = M/H$, i.e. by construction the highest momentum mode observed in the power spectrum is 'scaled' to $y_{0,max} = M/H$. The observed CMB power spectrum stretches to four orders of magnitude below that; it therefore ranges from $10^{-4}y_0$ to $y_0$.

For this maximal value of $y_0$ we see that the change in the power spectrum equals

$$\begin{aligned}\frac{P^{dS}_{BD} + \delta P}{P^{dS}_{BD}}(y_{0,max}) &= 1 + \frac{\pi}{4}\frac{\beta H}{M}\left[i\frac{M^2}{H^2}\bar{\mathcal{H}}^2_{3/2}(M/H) + \text{c.c.}\right] \\ &\simeq 1 + \beta\sin(2M/H) \ . \end{aligned} \tag{12}$$

Note: though the change in the power spectrum is parametrically $H/M$ as argued before (see eq. (9)), its maximal change is quite independent of their values — if one chooses $a_0 = k_{max}/M \leftrightarrow y_{0,max} = M/H$. For this specific value of $y_0$ the change in the power spectrum is linearly dependent on the size of the irrelevant operator $\beta$. We have drawn the change in the power spectrum in figure 1. The explicit breaking of de Sitter scale invariance by the leading irrelevant operator results in a linear momentum dependence of the amplitude of the oscillatory correction to power spectrum. This enhancement at high momentum means that for small values of $\beta$ and only moderately large values of $M/H$ the change in the power spectrum is far larger than the projected 1% uncertainty in future measurements. We have a solid case that for a large enough value of $H/M$ future CMB measurements are sensitive to high energy physics through irrelevant corrections to the initial conditions. Moreover, figure 1 clearly shows that the current sensitivity with which the power spectrum is measured already constrains the allowed values for $\beta$ and $H/M$ in nature. A coarse extrapolation from the WMAP results [24] indicates that the observed power spectrum is scale invariant with an accuracy of around 10%. A value of $\beta \sim 0.2$ and $H/M \sim 0.01$ would already imply a 20% change at the upper end of the power spectrum, inconsistent with the data. This establishes the point of principle that the power spectrum can be sensitive to irrelevant corrections. Recent data sensitivity studies confirm in much more detail that in specific scenarios the contribution of physics beyond $M$ can be disentangled from the data given a high-enough value of $H/M$ [10].

Other measurements, however, in particular the near absence of gravitational backreaction could constrain the size of the irrelevant operator beyond the direct measurement in the power spectrum. Indeed an order of magnitude estimate indicates that this will be so [12]. This is illustrated in the second panel of figure 1. The remainder of this article will show that a precise calculation of the gravitational backreaction reveals that the leading order of magnitude result is subject to a renormalization prescription. The physical second order result imposes weaker constraints on the size of irrelevant corrections. There is therefore a distinct *window of opportunity* to measure short distance physics in the CMB: the shaded region in figure 1.

### *2.2. Corrections to the Green's function*

To compute the one-loop correction to the gravitational stress-tensor, we will need the Green's function for $\phi$ with initial conditions set by the effective boundary coupling $\kappa_{eff}$ in eq. (8). Rather than adhering to the Lagrangian formalism, involving the parameter $\kappa$ to encode the initial state, it is instructive to translate back to the Hamiltonian formalism in which the Bogoliubov coefficients $b(k)$ parameterize the initial state. We do so to make contact with the standard approach in cosmology: details of the Lagrangian approach are discussed in [11].♯

Introducing two independent, homogeneous solutions $\varphi_{\pm}(k,\eta)$ of the bulk equations, the general solution is

$$\varphi_b(k,\eta) = \varphi_+(k,\eta) + b(k)\,\varphi_-(k,\eta)\ , \tag{13}$$

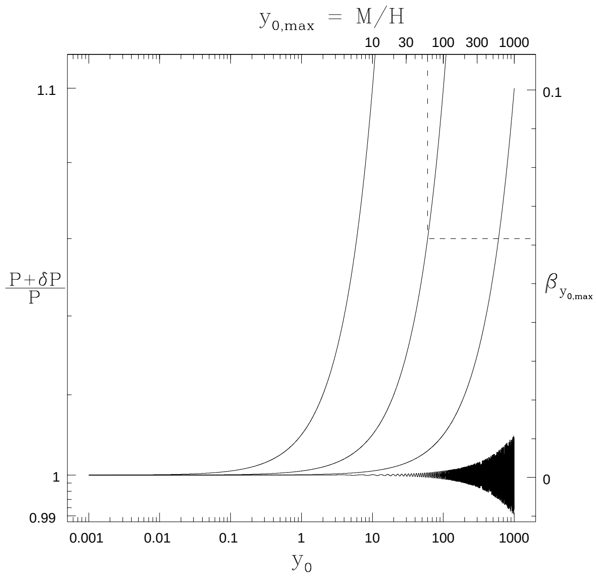
with $\varphi_- = (\varphi_+)^*$ and normalized according to the Klein-Gordon inner product (see eq. (19)).

Expanding the fields onto a basis of the independent solutions, and promoting them to operators,

$$\phi_b(x,\eta) = \int \frac{d^3k}{(2\pi)^3}\left[\hat{a}_b(k)\bar{\varphi}_b(k,\eta) + \hat{a}_b^\dagger(-k)\varphi_b(-k,\eta)\right]\, e^{i\vec{k}\cdot\vec{x}}\ , \tag{14}$$

one defines the vacuum as the state which is annihilated by $\hat{a}_b(k)\,|b\rangle = 0$. With respect to this vacuum, we then construct a Green's function from the time-ordered product of

♯ An extensive treatment of the Hamiltonian approach adiabatic order 4 vacua is forthcoming [25]. Here, as mentioned, we will be discussing non-adiabatic initial states.



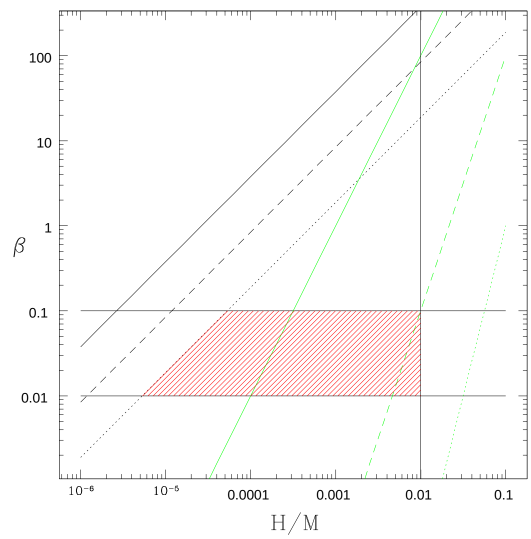




**Figure 1.** *A refined estimate of the sensitivity of the CMB to new physics.*
The left panel shows the change in the (amplitude of the) power spectrum due to the presence of the leading order irrelevant operator $\frac{\beta}{M}(\partial_i\phi)^2$ as a function of the physical momentum in units of the size of the horizon at the earliest time. (Only for one specific choice is the full oscillatory Bessel function behavior plotted.) This graph should be read as follows. Given the scale of new physics $M$ and the Hubble constant $H$ during inflation (or more precisely at the time when the highest mode $k_{max}$ of interest exits the horizon) the earliest time up to which we can trust the effective action is $y_{0,max} \equiv k_{max}/a_{0,min}H = M/H$ (see section 2.1). Anything to the right of $y_{0,max}$ should be discarded as untrustworthy. The observed CMB stretches to four orders of magnitude smaller momenta from $10^{-4}y_{0,max}$ to $y_{0,max}$. Precisely at $y_{0,max}$ the change in the power spectrum is linearly dependent on the value of $\beta$. The values of $M/H$ and $\beta$ corresponding to the various curves can thus be read off from the intersection of the plumblines to the upper and right axis. The right panel shows an exclusion plot for $\beta$ as a function of $H/M$. The $45^o$ lines correspond to the backreaction bounds derived in section 3 (continuous for zeroth order in slow roll, dashed for first order in slow roll, dotted for second order in slow roll). The $60^o$ lines correspond to the order of magnitude estimate for the backreaction [12]; they are equivalent to an estimate based on dimensional analysis. The upper horizontal line is an order of magnitude estimation of the current error to which we have a nearly scale invariant spectrum [24]. The lower horizontal line is an order of magnitude estimate of the cosmic variance limitations of resolution. Finally the vertical line denotes a maximal value of $H/M$ consistent with observation using a value for $H$ extracted from the allowed scalar/tensor ratio and $M \equiv 10^{16}$ GeV. This leaves the shaded region as the *window of opportunity* to observe transplanckian physics in the CMB.

fields.

$$G_b(k;\eta_1,\eta_2) \equiv \langle b|T(\phi_b(k,\eta_1)\phi_b(k,\eta_2)|b\rangle$$
$$= \frac{1}{(1-b\bar{b})}\left(\bar{\varphi}_b(k,\eta_1)\varphi_b(k,\eta_2)\theta(\eta_1-\eta_2)+(\eta_1\leftrightarrow\eta_2)\right) \ . \qquad (15)$$

Demanding that the Green's function obeys the boundary condition

$$a(\eta_0)^{-1}\partial_\eta G_b(k;\eta,\eta_2)\big|_{\eta=\eta_0} = -\kappa G_b(k;\eta_0,\eta_2)$$

inherited from eq. (6), one immediately realizes that this implies that the mode-function $\varphi_b$ obeys the same boundary condition as the field.†† It is then straightforward to deduce the following relation between the parameters $\kappa(k)$ and $b(k)$

$$b_\kappa(k) = -\frac{\kappa(k)\varphi_{+,0}+\partial_n\varphi_{+,0}}{\kappa(k)\varphi_{-,0}+\partial_n\varphi_{-,0}} \ , \qquad \varphi_{\pm,0}\equiv\varphi_\pm(\eta_0) \ . \qquad (16)$$

The vacuum state $|b_\kappa\rangle$ which is annihilated by $\hat{a}_{b_\kappa}(k)$ corresponds to the initial conditions in effective field theory set by the boundary coupling parameter $\kappa(k)$. The complicated relation between $b(k)$ and $\kappa(k)$ illustrates why the Hamiltonian framework is not well-suited to discuss renormalization and effective actions. The Hamiltonian theory has no natural expansion in irrelevant operators; here decoupling is not manifest in contrast to the Lagrangian framework.

The canonical de Sitter space Green's function is built on the preferred choice for the cosmological vacuum, the Bunch-Davies/adiabatic state. In the Hankel function $\mathcal{H}_\nu$ basis of de Sitter mode-functions, this vacuum is given by the choice $b(k)=0$:†

$$\varphi_+^{BD}(-k\eta) = (-k\eta)^{3/2}\sqrt{\frac{\pi}{4k}}\left(\frac{H}{k}\right)\bar{\mathcal{H}}_\nu(-k\eta) \ ,$$
$$\varphi_-^{BD}(y) \ = (\varphi_+^{BD})^*(y) \ , \quad \nu = \sqrt{\frac{9}{4}-\frac{m^2}{H^2}} \ . \qquad (17)$$

In this basis departures from the Bunch-Davies state are directly parameterized by a non-zero $b(k)$. Below we study small irrelevant deformations of the BD initial conditions.

††The Green's function is a solution to a second order differential equation and therefore requires two boundary conditions to determine it uniquely. The other boundary condition in this case is $\partial_n G|_{\eta=far\ future} = -\bar{\kappa}G$. Chapter 6 of [21] explains why for the dominant part of the gravitational backreaction we can choose the in- and out-states the same. How to impose two independent boundary conditions in the future and the past is explained in [11].

† Translating back to Lagrangian effective field theory, eq. (16) shows that the BD-state corresponds to a momentum-dependent effective boundary coupling $\kappa_{BD}(k)$. The fact that the the BD-state by definition reduces to the flat Minkowski vacuum for high $k$ does mean that all irrelevant boundary operators are zero for the BD-state. There are strong indications that the BD initial conditions is an IR-fixed point of boundary RG-flow. (Qualitatively this is suggested by the fact that the BD state is the adiabatic vacuum. Quantitatively it is known that the BD-state is a fixed-point of the 'shift'-symmetry inherent in all boundary actions [11].) This would mean that they are preferred initial conditions from an effective field theory perspective as well. For an IR fixed point it is a sensible procedure to study small departures by turning on irrelevant operators (8). We proceed on this assumption.

The non-zero contribution to $b(k)$ corresponding to the leading irrelevant deviation $\delta\kappa = \beta\frac{k^2}{a_0^2 M}$ from the BD initial state $\kappa_0 = \kappa_{BD}$, can be obtained by expanding (16) to lowest order in $\delta\kappa$.

$$\delta b(k) = -\delta\kappa(k)\left(\frac{(\varphi_{+,0}^{BD})^2}{\varphi_{+,0}^{BD}\left(\kappa_{BD}\varphi_{-,0}^{BD} + \partial_n\varphi_{-,0}^{BD}\right)}\right) + \mathcal{O}(\delta\kappa^2)\ . \tag{18}$$

Using (16) (with $b_{BD}=0$) to solve for $\kappa_{BD} = -\partial_n\varphi_+^{BD}/\varphi_+^{BD}$, the denominator in this expression can be rewritten using the Klein-Gordon normalization condition for the dS mode functions $\varphi_\pm$

$$\varphi_+\,\partial_n\varphi_- - \varphi_-\,\partial_n\varphi_+ = ia_0^{-3}\ , \tag{19}$$

With these relations we obtain the following expression for $\delta b(k)$

$$\delta b(k) = ia_0^3\,(\varphi_{+,0}^{BD})^2\,\delta\kappa = ia_0^3\,(\varphi_{+,0}^{BD})^2\left(\beta\frac{k^2}{a_0^2\,M}\right)\ . \tag{20}$$

Note that the boundary coupling $\kappa$ is independent of the choice of basis $\varphi_\pm$ but $b$ and hence $\delta b$ is not. Let us also emphasize that this expression can only be trusted for small $\delta\kappa$, i.e. for boundary physical momentum scales $p_0 = k/a_0$ smaller than the cut-off scale $M$.

The change in the Green's function eq. (15) due to the deviation $\delta b_{\delta\kappa}(k)$ from the BD state is now readily determined,

$$\begin{aligned}
G_{\delta b}(k,\eta_1,\eta_2) \quad &= G_0(k,\eta_1,\eta_2) \\
&\quad + \Big[\Big(\delta b\,\varphi_-^{BD}(\eta_1)\varphi_-^{BD}(\eta_2) + \text{c.c.}\Big) \\
&\qquad + 2|\delta b|^2\,\varphi_-^{BD}(\eta_1)\varphi_+^{BD}(\eta_2)\Big]\theta_{12} + (\eta_1\leftrightarrow\eta_2) + \mathcal{O}(\delta b^3) \\
&= G_0(k,\eta_1,\eta_2) \\
&\quad + \Big[\Big(\left(ia_0^3(\varphi_{+,0}^{BD})^2\delta\kappa + \mathcal{O}(\delta\kappa^2)\right)\varphi_-^{BD}(\eta_1)\varphi_-^{BD}(\eta_2) + \text{c.c.}\Big) \\
&\qquad + 2a_0^6|\varphi_{+,0}^{BD}|^4|\delta\kappa|^2\varphi_-^{BD}(\eta_1)\varphi_+^{BD}(\eta_2)\Big]\theta_{12} + (\eta_1\leftrightarrow\eta_2) + \mathcal{O}(\delta\kappa^3)\ .
\end{aligned} \tag{21}$$

We expanded to *second* order in $\delta b$. Contrary to expectation — as we will show in the next section — it is the $a_0^6|\delta\kappa|^2$ term in the above equation whose contribution dominates the one-loop gravitational backreaction. (We will not need to know the exact $\delta\kappa^2\varphi_-^2$ + c.c. terms, despite formally being of the same order.)

## 3. Backreaction from initial state corrections

The one-loop backreaction we will calculate is formally a divergent quantity. There are three important points to make in that regard — one general and two specific:

*a)* In cosmological settings the Hadamard constraint that the stress-tensor can be rendered finite by subtracting the flat space counterterm has long been used as an initial state selection rule. In [11] we argued that there are a large number of non-Hadamard initial conditions for which the stress tensor can be consistently renormalized. Here we will focus on the specific irrelevant correction (7) to the Hadamard BD-state.

*b)* Wishing to discuss effects of new physics which are encoded in irrelevant *non-renormalizable* operators, an explicit cut-off is required to maintain finiteness of the theory. 'Renormalization' is the redefinition of all quantities under a change of this cut-off, such that the new theory reproduces the same physics (see e.g. [13]).

*c)* In field-theoretic language the constraint that the expectation value of the stress-energy tensor can be rendered finite is equivalent to renormalization of the composite operator $T_{\mu\nu}$. Inherently any renormalization also needs a renormalization prescription to determine the finite remainder after the subtraction of divergences. This prescription amounts to setting a predetermined correlation function equal to an experimentally measured quantity at a chosen scale.‡ In a theory with a boundary action the stress tensor consists of two parts; a bulk and a boundary contribution. Generically each will be divergent and each will need a separate renormalization prescription. The boundary stress tensor is naturally fixed by the value of $T_{\mu\nu}$ on the fixed conformal time boundary at $\eta_0$. This quantity is beyond experimental reach, and the renormalization prescription to use will be unknown and ambiguous. However, the "boundary-energy" encoded in the boundary stress tensor is localized; the bulk physics far from the boundary is insensitive to the particular boundary renormalization prescription. The theory is predictive.

Recall also that 1) interactions are not relevant for the one-loop contribution to the stress tensor and that 2) we may limit our attention to a massless scalar field as the high $k$ modes running around the loop will dominate the answer (at the cost of an IR divergence, which we should take care to isolate). With point *c)* above in mind, the computation therefore reduces to the expectation value of the bulk stress tensor for a free massless scalar field w.r.t. the vacuum $|b\rangle$:

$$\begin{aligned} \langle b|T_{\mu\nu}(\eta)|b\rangle &= \langle b|T^{bulk}_{\mu\nu}|b\rangle \ , \qquad \mathit{if} \ \ \ln\left|\frac{\eta_0}{\eta}\right| \gg \frac{H}{M} \ , \\ T^{bulk}_{\mu\nu} &= \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\, g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi \ . \end{aligned} \tag{1}$$

By neglecting the contribution of the boundary stress tensor, the validity of this expression is intrinsically limited to (physical) distances at least one cut-off length $1/M$ away from the boundary (see e.g. [11]). In conformal time this implies the bound stated above.

Finally, in a homogeneous and isotropic background the two non-zero components of the stress tensor are the density $T_{\eta\eta} = a^2(\eta)\rho$ and the pressure $T_{ij} = g_{ij}p$. Expressed in terms of the useful quantities

$$K_{\mu\nu} \equiv \partial_\mu\phi\partial_\nu\phi \ , \qquad \bar{K} = g^{ij}K_{ij} \ , \tag{2}$$

‡ Except for wavefunction renormalization, which is fixed by unitarity.

they are

$$\begin{aligned} a^{-2}T_{\eta\eta} &= \tfrac{1}{2}\left(a^{-2}K_{\eta\eta}+\bar{K}\right) &= \rho \ , \\ g^{\mu\nu}T_{\mu\nu} &= a^{-2}K_{\eta\eta}-\bar{K} &= -\rho+3p \ . \end{aligned} \tag{3}$$

We are now ready to evaluate the one-loop contribution to the stress-tensor. At one-loop the expectation value of the quantities $a^{-2}K_{\eta\eta}$ and $\bar{K}$ is given in terms of the *equal time* Green's function eq. (15)

$$\begin{aligned} \langle b|a^{-2}(\eta)K_{\eta\eta}(\eta)|b\rangle &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{a^2}\partial_{\eta_1}\partial_{\eta_2}G_b(k,\eta_1,\eta_2)\Big|_{\eta_1=\eta_2=\eta} \\ \langle b|\bar{K}(\eta)|b\rangle &= \int \frac{d^3k}{(2\pi)^3}\frac{k^2}{a^2}G_b(k;\eta,\eta) \ . \end{aligned} \tag{4}$$

Expanding the Green's function around the BD-state to second order as in eq. (21) each will have three contributions.

### *3.1. Zeroth order BD term*

The zeroth order term is the one-loop expectation value of the stress-tensor w.r.t. the BD vacuum [21]. We will not re-compute it here. What is important for us, is to note that the naively divergent part is *independent* of $\eta$, if the regularization procedure is so. This follows from the fact that the BD-state is Hadamard. This constant divergence can be cancelled by the Minkowski space counterterm plus a finite *constant* piece adjusted according to the renormalization prescription. As is well known, there are small finite time-dependent remainders. This is the 'perturbative' quantum instability of de Sitter space [26]. Compared to the effect of the irrelevant correction to the Bunch-Davies vacuum, these terms will be negligible.

### *3.2. First order term*

A priori we expect the linear term in $\delta b$ to be the leading correction to the stress tensor. The first order correction to the Green's function, however,

$$\Delta^{(1)}G(k;\eta_1,\eta_2)\Big|_{\eta_1=\eta_2=\eta} = \left(ia_0^3\delta\kappa(\varphi_{+,0}^{BD})^2(\varphi_{-}^{BD}(\eta))^2+\text{c.c.}\right) \tag{5}$$

is a highly oscillating function in $k$. Indeed this oscillatory behavior is a distinct characteristic of corrections to the BD-vacuum, which may make them experimentally identifiable in the inflationary power spectrum. The backreaction, on the other hand, corresponds to the integral of the Green's function over $k$ and here the oscillatory peaks and valleys will largely cancel each other out. For a massless field the integrals can be done exactly, but the qualitative effect is clearly illustrated by the dominant high $k$ part of the integral. In this limit the mode functions simplify to

$$k \gg -\frac{1}{\eta} \qquad \varphi_+(k,\eta) = -\frac{1}{\sqrt{2ka^2}}e^{ik\eta}+\mathcal{O}(k^{-3/2}) \tag{6}$$

resulting in a simple sine for the first order Green's function [12, 15]

$$k \gg -\frac{1}{\eta} \qquad \Delta^{(1)}G(k;\eta,\eta) = ia_0\frac{\beta k^2}{M}\frac{1}{4k^2a_0^2a^2}e^{2ik(\eta_0-\eta)} + \text{c.c} + \ldots$$
$$= \frac{\beta}{2Ma_0a^2}\sin(2k(\eta-\eta_0)) + \ldots \tag{7}$$

In this high $k$ approximation the quantity $\bar{K}$, regulated with a smooth cut-off function, evaluates to

$$\bar{K} \sim \frac{\beta}{Ma_0a^4}\int dk k^4 \sin(2k(\eta-\eta_0))e^{-k^2/2\mathcal{M}^2}$$
$$\sim \frac{\beta}{Ma_0a^4}\mathcal{M}^5 e^{-2\mathcal{M}^2(\eta-\eta_0)^2}\Big(1+\ldots\Big)\ . \tag{8}$$

Note that the answer decreases in time only for a comoving cut-off $\mathcal{M} = a_0M$ rather than a physical one $\mathcal{M} = a(\eta)M$; we will comment on this below. The main point is that in terms of physical time $t = -\frac{1}{H}\ln|H\eta|$, $\bar{K}$ damps with scale $\mathcal{M}/a_0 = M$.

$$\bar{K} \sim \beta M^4\frac{a_0^4}{a^4}e^{-2\frac{M^2}{H^2}(1-\exp(-H(t-t_0))^2)} + \ldots$$
$$\sim \beta M^4\frac{a_0^4}{a^4}e^{-2M^2(t-t_0)^2(1+\mathcal{O}(H(t-t_0)))} + \ldots \tag{9}$$

The exact answer for $\bar{K}$ (see figure 2) shows that all significant terms observe this scaling behavior.§ This result has the important consequence that the first order backreaction, though non-vanishing, is essentially fully localized on the boundary. Recall that in a low-energy effective theory with cut-off scale $\mathcal{M}$ all objects are smeared out over this scale: all boundary objects effectively behave as a regulated distribution with scale $M$ — Gaussian $Be^{-\mathcal{M}^2(t-t_0)^2}$ or otherwise. We have therefore ventured outside the range of validity of using only the bulk stress tensor to compute the backreaction (1). The neglected formally infinite boundary stress tensor and its counterterm contribute at first order with the same scaling behavior. The first order term is therefore almost completely fixed by the boundary renormalization prescription, and has no significant bulk remnant.

We conclude that the most relevant term *for the bulk backreaction* is the first non-oscillatory contribution in the integral: the second order $a_0^6\delta\kappa^2$ term mentioned below eq. (21). This had already been surmised by Tanaka [15].‖ We note that in [12] the first

§ Because the Bessel function part of the mode functions simplify for a massless field ($H_{3/2} = -\sqrt{\frac{2}{\pi z}}e^{iz}(1+i/z)$) the exact answer with a Gaussian cut-off is readily obtained in terms of error-functions. Expanding around $M=\infty$, the first term which fails to scale as $e^{-M^2t^2}$ scales inversely with $M$:

$$\bar{K} = \beta\left[e^{-M^2t^2}M^4\left(1+\mathcal{O}(\frac{H}{M})\right) + f(t)\frac{H}{M}\left(1+\mathcal{O}(\frac{H}{M})\right)\right] + \mathcal{O}(\beta^2)$$

where $f(t)$ is a polynomial function in $Ht$.

‖ Formally this is of the same order as the first subleading irrelevant operator correction $\beta_2k^3/M^2(\phi_{+,0}^2\phi_-^2 + \text{c.c.})$. As is clear from eq. (21), however, its contribution will also be oscillatory and localized to the boundary.

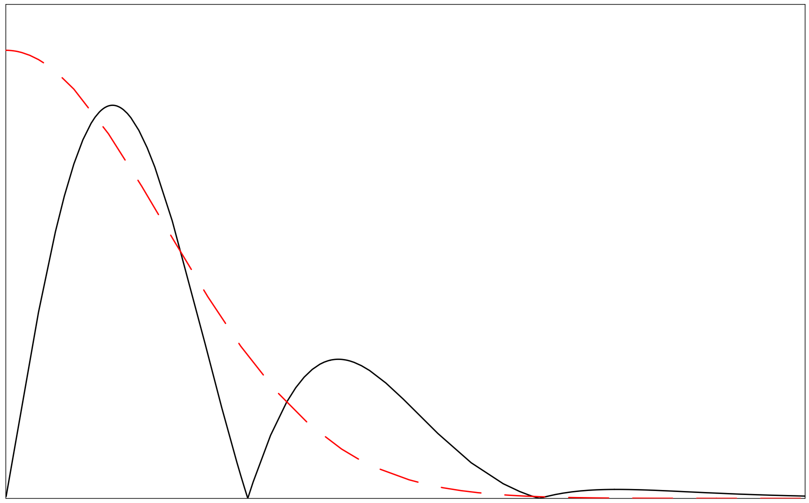


**Figure 2.** Exact first order correction to stress-tensor due to the irrelevant operator $\beta$ (absolute value of $\bar{K}$ in units of the classical density $\rho = 3H^2M_{pl}^2$; solid) compared to the exponential scaling $e^{-2M^2(t-t_0)^2}$ (dashed) derived in the high $k$ approximation.

order term is used to put constraints on the coefficient $\beta$. Clearly first order constraints are formally much stronger, but their time-dependence indicates that they are subject to a renormalization ambiguity.

*Which cut-off?* Before we compute this leading non-oscillatory term, we give a qualitative explanation why the cut-off used ought to be $\mathcal{M} = a_0 M$ rather than $\mathcal{M} = a(\eta)M$. In general one is always free to use whatever cut-off function one fancies, provided it renders all Green's functions finite. In practice one chooses one that respects the most symmetries of the theory: this guarantees that the Ward identities are manifestly obeyed in the regulated theory. A canonical example for a regulator function is to modify the kinetic term to

$$-\frac{1}{2}\int \partial_\mu\phi\partial^\mu\phi \to -\frac{1}{2}\int e^{-\frac{D_\nu D^\nu}{M^2}}\partial_\mu\phi\partial^\mu\phi \tag{10}$$

where $D_\mu$ is the covariant derivative. In a cosmological setting the metric inherent in the contraction $D_\mu D^\mu$ introduces a time-dependence in the cut-off function for the (spatial) momenta: $\exp(-\frac{D_iD^i}{M^2}) = \exp(k^2/a^2M)$. What matters for the computation above is how this cut-off function affects the Green's function. Recall that the Green's function is a bi-local function of two points, whereas the regulating function is well-defined (in terms of derivatives) at one point. A qualitative way to understand better what is happening is to not modify the kinetic term explicitly, but rather implicitly through a change in the fields

$$\phi(x) \to e^{-\frac{D_\mu D^\mu}{2M^2}}\phi. \tag{11}$$

This suggests a (qualitative) change for the cosmological mode functions

$$\varphi(k,\eta) \to e^{-\frac{k^2}{2a^2M^2}}\varphi(k,\eta) \tag{12}$$

The Green's function is therefore modified to

$$G(k;\eta_1,\eta_2) \rightarrow e^{-\frac{k^2}{2a_1^2M^2}} G(k;\eta_1,\eta_2) e^{-\frac{k^2}{2a_2^2M^2}} \tag{13}$$

Inferring from the relation between the stress-tensor expectation value and the Green's function, $\langle b|T_{\mu\nu}(\eta)|b\rangle \sim \partial_{\mu_1}\partial_{\nu_2}G(k;\eta_1,\eta_2)|_{\eta_1=\eta_2}$, one is inclined to use a naive regulating function $e^{-k^2/a^2M^2}$ in eq. (8). This is *not* correct. When the change in the boundary condition is treated as a perturbation, the first order term involves not one but two Green's functions; each interpolates between a boundary vertex and the location of the composite operator. This is clearly shown in [11] and is indicated by the presence of four mode functions in eq. (5). The counterintuitive answer obtained for $\bar{K}$ with a physical cut-off $\mathcal{M} = a^2M^2$ is a manifestation of using this wrong cut-off.

According to the above, it is clear that the regulating function to be used in eq. (8) is

$$\mathcal{F}_{reg} = e^{-\frac{k^2}{a^2M^2} - \frac{k^2}{a_0^2M^2}} = e^{-\frac{k^2}{a_0^2M^2}\left(\frac{a^2/a_0^2+1}{a^2/a_0^2}\right)} \simeq e^{-\frac{k^2}{a_0^2M^2}} . \tag{14}$$

The time-dependence in the ratio $\frac{a^2/a_0^2+1}{a^2/a_0^2}$ which smoothly interpolates between two and unity is clearly not relevant.

### *3.3. Second order term*

Let us now proceed and calculate the dominant, non-oscillatory, second order contribution to the backreaction (1). In the high $k$ approximation (6) for the mode functions the integral can easily be performed and after some straightforward algebra, we find

$$\begin{aligned}
\bar{K}^{(2)} &= \frac{1}{3(4\pi)^2}\,|\beta|^2\,M^4\,e^{-4H(t-t_0)}\left[1+\mathcal{O}\left(e^{2H(t-t_0)}\frac{H^2}{M^2};\frac{H}{M}\right)\right] \\
a^{-2}K^{(2)}_{\eta\eta} &= \frac{1}{3(4\pi)^2}\,|\beta|^2\,M^4\,e^{-4H(t-t_0)}\left[1+\mathcal{O}\left(e^{2H(t-t_0)}\frac{H^2}{M^2};\frac{H}{M}\right)\right] .
\end{aligned} \tag{15}$$

The equality of the two leading terms, implying $\rho = 3p$, reflects the absence of a scale in a free massless field. They differ of course at the subleading level, as an exact calculation shows.

The time behavior of the answer is fundamentally different from the first order term (9). This can be attributed to the non-oscillatory behavior of the integrand. The backreaction still decays exponentially fast away from the position of the boundary, but now with the Hubble scale $H$ rather than the cut-off scale $M$. As this contribution is clearly unaffected by both the renormalization prescription of the bulk stress tensor (a constant time-independent term) and the boundary stress tensor (a localized term scaling with the cut-off $\mathcal{M}$), it unmistakably corresponds to a physical contribution. Hence an irrelevant perturbation away from the BD-state introduces an excess amount of physical energy density [8]. As time progresses this energy density red-shifts due to the dS exponential expansion, explaining the exponential decay with scale $H$. Because this contribution entails a physical change in the background, the observed cosmology constrains the size the coefficient $\beta$ of the irrelevant operator. The question is whether these constraints (dis)allow potentially observable corrections to the CMB.

### *3.4. Adiabaticity and initial conditions*

Aside from determining phenomenological constraints on possible high-energy physics effects in the CMB, the calculation of the one-loop backreaction elucidates a theoretical argument as well. By consensus the preferred vacuum state in cosmological settings is the adiabatic one. This state is defined as that state in which the number operator decreases slowest with time. This is a somewhat uncomfortable definition as the number operator is not an observable quantity. Its observable counterpart, however, is the normal-ordered Hamiltonian, i.e. the renormalized stress tensor. An improved definition of the adiabatic initial conditions is those for which the (one-loop) stress-tensor changes slowest in time. As our calculation clearly shows, any irrelevant perturbation away from the BD-initial conditions will introduce a scaling $e^{-nH(t-t_0)}$. Hence we recover that the BD-initial conditions are the adiabatic ones. Moreover irrelevant corrections to the BD-state quantitatively parameterize deviations away from adiabaticity.

This quantitative understanding of non-adiabatic initial states (indeed the presence of irrelevant operators means that the theory is not renormalizable and decoupling no longer works exactly) and the consequent time-dependence of the stress-tensor illustrates an interesting new contribution to the standard slow-roll inflationary scenario (see the recent article [27] for a qualitatively similar viewpoint). As a variant of one-loop induced inflationary potentials, the above computation shows how non-adiabaticity can be responsible for deviations from the classically expected scale-invariant de Sitter spectrum. Such non-adiabatic contributions to inflationary evolution are arguably natural in the interpretation of cosmological evolution as relaxation towards a ground state. Experimental evidence, however, ought to judge the full validity of such a 'relaxation'-scenario. In the next section we will discuss to what extent deviations from adiabaticity, i.e. the contribution to the power spectrum from boundary irrelevant operators, contribute to the inflationary evolution and are constrained by current observation.

## 4. Constraints from backreaction

These constraints will follow from comparing the measured vs. predicted gravitational background. We have already shown the results of this calculation in the right panel of Figure 1 where we compare them with the direct observational bounds from the power spectrum plus a constraint on the maximal value of $H/M$. Let us now show, how we arrived at the bounds in Figure 1.

The Friedmann equation relates the measured full quantum corrected Hubble scale $H_{eff}$ to the expectation value of the stress-tensor

$$H_{eff}^2(\eta) = \frac{1}{3M_p^2}\langle T_{00}\rangle = \frac{1}{3M_p^2}\left(T_{00}^{(0;0)}(\eta) + T_{00}^{(1;0)}(\eta) + T_{00}^{(1;1)}(\eta) + T_{00}^{(1;2)}(\eta) + \ldots\right) , \quad (1)$$

Here $T_{00}^{(n;m)}$ corresponds to the $n$-loop order $\beta^m/M^m$ irrelevant corrections to the backreaction, and $M_p$ equals the reduced Planck mass $M_p \approx 2.4 \cdot 10^{18}$ GeV. We have shown earlier that $T_{00}^{(1,1)}$ is localized on the boundary and, concentrating only on the leading $M^4$ contributions, that the $T_{00}^{(1,0)}$ term is constant at this order and fixed by the renormalization prescription. We will therefore ignore their contributions from here on. $T^{(0,0)}$ is of course the classical background and at leading order $M^4$, $T_{00}^{(1,2)}$ follows from (15)

$$T_{00}^{(1,2)} = \frac{1}{48\pi^2}|\beta|^2 M^4 e^{-4H_0(t-t_0)} \tag{2}$$

with $H_0$ the classical Hubble scale.

The first constraint is simply that of a consistent perturbation theory, i.e. that the one-loop backreaction term $T_{00}^{(1,2)}$ should be small compared to the measured expansion rate of the universe

$$\frac{T_{00}^{(1,2)}}{T_{00}^{eff}} \ll 1 \quad \Rightarrow \quad |\beta|^2 e^{-4H_0(t-t_0)} \ll (12\pi)^2 \left(\frac{M_p^2 H_{eff}^2}{M^4}\right) . \tag{3}$$

This constraint is strongest at the 'earliest time' $t \sim t_0$ where we set the initial conditions. We see that $|\beta|^2 \ll (12\pi)^2 M_p^2 H^2/M^4$. In conventional string scenario, the string scale is roughly two orders of magnitude below the (reduced) Planck scale which gives $|\beta|^2 \ll 10^3$ for $H \sim 10^{14}$ GeV. This establishes our claim that the backreaction constraint has no practical content for $H/M > 10^{-4}$. With the numbers used for $H$ and $M$ the power spectrum already constrains $\beta \leq 1$. The reason why this is so, is also evident. The weakness of the backreaction constraint is a result of the cancellation of the highly oscillatory integrand. The power spectrum, however, is roughly the integrand of the one-loop backreaction. For the power spectrum, the oscillations do not cancel each other, which explains the higher sensitivity compared to the backreaction constraint.

In addition to this 'static' backreaction constraint, one should also demand that the time derivatives of the backreaction are not too big. These are the phenomenological parameters that determine the characteristics of the inflationary evolution. Through one-loop backreaction the boundary irrelevant operators will contribute to these, as we anticipated in section 3.4. The derivatives must remain small enough, however, to guarantee that inflation lasted long enough and to explain the measured scale-invariance of the spectrum. The (first) time-derivative of the stress tensor is related to the conventional inflationary slow-roll parameter $\epsilon \equiv -\frac{\dot{H}}{H^2}$ as $H\dot{T}_{00}/T_{00} = -2\epsilon$. To guarantee inflation $\epsilon$ should be smaller than 1

$$-\frac{\dot{T}_{00}}{6H_{eff}^3 M_p^2} \equiv \epsilon_{eff} \ll 1 \quad \rightarrow \quad \frac{1}{2(6\pi)^2}|\beta|^2 \frac{M^4}{M_p^2 H^2} e^{-4N} \simeq \epsilon_{eff} - \epsilon^{(0)} . \tag{4}$$

For the classical de-Sitter background used here $\epsilon^{(0)} = 0$ which implies the following constraint on $\beta$ (again evaluated close to the boundary)

$$|\beta|^2 \lesssim 2(6\pi)^2 \left(\frac{M_p^2 H_0^2}{M^4}\right) |\epsilon_{eff}| . \tag{5}$$

Using the same estimates as before, plus the experimental value for $\epsilon_{eff} \sim 10^{-1}$ we conclude that $|\beta|^2 \lesssim 10^2$, which is again superseded by the power spectrum sensitivity itself.

Finally the measured scale invariance of the power spectrum constrains the second time derivative of the backreaction.¶ It is related to a combination of the first and the second slow-roll parameter $\frac{\ddot{T}_{00}}{H^2 T_{00}} = 2\epsilon\eta + 2\epsilon^2$. We deduce

$$\frac{\ddot{T}_{00}}{2H^2 T_{00}} \equiv \epsilon_{eff}(\eta_{eff} + \epsilon_{eff})$$

$$\rightarrow \quad \frac{1}{2(3\pi)^2}|\beta|^2 e^{-4N}\left(\frac{M^4}{M_p^2 H_0^2}\right) \simeq \epsilon_{eff}(\epsilon_{eff} + \eta_{eff}) - \epsilon^{(0)}(\eta^{(0)} + \epsilon^{(0)}) \qquad (6)$$

For a general slow-roll inflationary background this will lead to the following constraint on $\beta$,

$$|\beta|^2 \lesssim (6\pi)^2 \left(\frac{M_p^2 H_0^2}{M^4}\right) \epsilon_{eff}(\epsilon_{eff} + \eta_{eff}) \ . \qquad (7)$$

This is clearly our strongest constraint. Using the same estimates as before and in addition assuming that $|\epsilon_{eff}| \sim 0.1$ , $|\eta_{eff}| \sim 0.1$, we approximately find $|\beta|^2 \ll 10$, but this number could vary somewhat depending on the precise estimates. Yet the power spectrum constraint is again stronger.

Eqs (3)- (7) are the bounds we have drawn in the right panel of figure 1. For comparison we have also shown the order of magnitude estimates made in [12]. Mathematically the latter do correspond to the first order one-loop correction to the backreaction. We have shown, however, how this contribution is localized on the boundary where one sets the initial conditions. It is therefore subject to a renormalization ambiguity. The second order contribution is physical but its constraints are a lot milder and are not threatening the potential observability of initial state corrections in the CMB. They clearly leave a large window of opportunity for a significant range of values of $H/M$ where irrelevant corrections will affect the CMB.

### *4.1. The length of inflation and initial conditions*

Precisely the fact that non-Bunch-Davies initial conditions ought to correspond to a quantum physical contribution to the stress tensor has been used as a qualitative argument that irrelevant corrections to the initial conditions cannot be large enough to be potentially observable. The energy present in the initial state would *blueshift* towards the past and rapidly invalidate the classical inflationary background used. Since we need roughly sixty e-folds of inflation to explain the observed flatness and isotropy of our universe this would be problematic. With the quantitative boundary effective formalism to encode the initial conditions we can investigate this issue more deeply.

¶ Because we only have observational knowledge about the first and second order slow roll coefficients, higher derivatives $d^n T_{00}/dt^n$ place no phenomenological constraints on the value of $\beta$. An observed running of the spectral index would bound the third derivative as well.

Indeed the scaling with $H$ of the second order correction to the stress tensor implies that for nonzero $\beta$ there is an excess physical energy present on top of the classical background. This backreaction starts to dominate over the classical background when

$$e^{4N} = (12\pi)^2 \left(\frac{M_p^2 H^2}{M^4|\beta^2|}\right) \tag{8}$$

with $N = H(t_0 - t)$ the total number of e-folds *before* the 'earliest time'. Only for very small values of $H/M$ is this number

$$N \simeq \frac{21}{4} - \frac{1}{2}\ln|\frac{\beta M}{H}| \ . \tag{9}$$

negative. For the range of values $H/M$ of interest the time where backreaction starts to dominate is therefore *before* the 'earliest time' where one can start to trust the cosmological low energy effective action. What truly happens before the 'earliest time' is unanswerable within the low energy effective framework. This does not mean that the energy that is stored in the initial state miraculously disappears or is not an backreaction issue before time $t = t_0$. It does mean that we need to know the short distance completion of the theory to answer this criticism. Within the range of validity of the effective action after the 'earliest time' the backreaction is always a small perturbation on the classical background. The cosmological low energy effective action with boundary is consistent.

A separate criticism argued that irrelevant corrections to the initial conditions were necessarily fine tuned. We will answer this elsewhere; a first estimate is made in [28]. One noted fine tuning objection: that any momentum dependent feature in the power spectrum must be fine tuned as we only measure a narrow ten $e$-fold window of the full power spectrum produced during inflation, does not hold here. It is a straightforward exercise to show that any irrelevant operator will introduce a power dependence in the spectrum. Moreover, following the precepts of effective action this momentum dependence occurs in a well-defined derivative expansion which predicts outwardly differing but intrinsically universal results for any ten e-fold window of the power spectrum.

## 5. Conclusions

The main conclusion stemming from this work is that the unambiguous backreaction constraints on the irrelevant operator coefficient are mild and do not affect the potential observability of the initial state corrections in the CMB. The three constraints arising from the observed slow-roll inflationary phase read

$$(1) \quad |\beta|^2 \ll (12\pi)^2 \left(\frac{M_p^2 H_0^2}{M^4}\right) \ , \tag{1}$$

$$(2) \quad |\beta|^2 \lesssim \ 2\,(6\pi)^2\,|\epsilon_{eff}|\,(\left(\frac{M_p^2 H_0^2}{M^4}\right) \ , \tag{2}$$

$$(3) \quad |\beta|^2 \lesssim \ (6\pi)^2\,|\epsilon_{eff}|\,(|\eta_{eff}| \ \left(\frac{M_p^2 H_0^2}{M^4}\right) \ . \tag{3}$$

Typical, although slightly optimistic, estimates imply that $M_pH_0/M^2 \gtrsim 1$ and $\epsilon_0 \sim \eta_0 \lesssim 10^{-1}$. The constraints on $|\beta|^2$ thus range from $\mathcal{O}(10^3)$, $\mathcal{O}(10^2)$ to $\mathcal{O}(10)$. These numbers could easily change by an order of magnitude by slightly changing the precise parameter estimates, so there is some room for adjustment in these constraints. This is plotted in Figure 1. Nevertheless, using the suggested parameter estimates, order one coefficients for $\beta$ are still allowed by these constraints. Indeed the observability of the initial state corrections in the CMB is not ruled out. Rather the power spectrum itself places stronger constraints on $\beta$.

We have moreover shown that the irrelevant corrections to the boundary effective action encoding the initial conditions parameterize deviations from adiabaticity. They correspond to having an excited energetic initial state. Though this energy could cause a problem when blueshifted to the past, within the framework of the low energy effective action it is always perturbative. This ensures consistency of the cosmological low energy effective action including high energy induced irrelevant corrections to the initial state.

The burning question is whether these short distance corrections are actually decipherable from CMB data. Similar to earlier predictions of the contribution to the power spectrum, we find that the leading irrelevant boundary operator yields a characteristic oscillatory signature. In principle this ought to make these corrections uniquely identifiable in the data. Sensitivity studies in a specific UV-complete framework appear to bear this out [10]. As figure 1 shows, however, the leading irrelevant correction alone induces a very rapid oscillation period for those modes where the amplitude is large enough. From eq. (9) we see that its frequency is about $\omega \simeq \frac{y_0}{2\pi} \simeq \frac{kM}{2\pi k_{max}H} \sim \frac{M}{H}$. This is probably too rapid to be resolved in the actual data. It is difficult to make a concrete statement, though. In the end only a full deconvolving of the actual measured data with respect to the template provided here, can yield an answer to the question whether the accuracy with which the data is collected can uniquely identify the contributions from short distance corrections to the initial state. This is a calculation we intend to return to.

Let us finally mention that it would be extremely interesting if one could calculate the coefficients $\beta$ from first principles. For this we need to know the UV completion of the theory. For instance, it could be interesting to study some of the recently proposed string inflationary scenarios [29] to see whether one could explicitly calculate the coefficients of some of the irrelevant operators in this context. With a UV-completion in hand, one could moreover compute the unambiguous answer for the first-order backreaction. If that were possible one could start making concrete predictions for stringy signatures in the CMB spectrum, a particularly exciting prospect.

## Acknowledgments

It is a pleasure to thank Jan de Boer, Daniel Chung, Csaba Csáki, Richard Holman, Emil Mottola, Bob McNees, Massimo Porrati and Larus Thorlacius for interesting discussions and comments. GS and JPvdS are particularly grateful to the organizers and participants of the MCTP Dark side of the Universe May 2004 workshop and those of the Strings at CERN July 2004 workshop. KS and GS would like to thank the organizers and participants of the Banff New Horizons in String Cosmology June 2004 workshop. BRG and KS acknowledge financial support from DOE grant DE-FG-02-92ER40699. The work of GS was supported in part by NSF CAREER Award No. PHY-0348093, DOE grant DE-FG-02-95ER40896 and a Research Innovation Award from Research Corporation.

## References

[1] J. Martin and R. H. Brandenberger, *A cosmological window on trans-Planckian physics*, arXiv:astro-ph/0012031.

[2] T. Padmanabhan, *Acceptable Density Perturbations From Inflation Due To Quantum Gravitational Damping*, Phys. Rev. Lett. **60**, 2229 (1988).
T. Padmanabhan, T. R. Seshadri and T. P. Singh, *Making Inflation Work: Damping Of Density Perturbations Due To Planck Energy Cutoff*, Phys. Rev. D **39**, 2100 (1989).
R. H. Brandenberger, *Inflationary cosmology: Progress and problems*, arXiv:hep-ph/9910410.
J. Martin and R. H. Brandenberger, *The trans-Planckian problem of inflationary cosmology*, Phys. Rev. D **63**, 123501 (2001) [arXiv:hep-th/0005209].
R. H. Brandenberger and J. Martin, *The robustness of inflation to changes in super-Planck-scale physics*, Mod. Phys. Lett. A **16**, 999 (2001) [arXiv:astro-ph/0005432].
J. C. Niemeyer, *Inflation with a high frequency cutoff*, Phys. Rev. D **63**, 123502 (2001) [arXiv:astro-ph/0005533].

[3] R. Easther, B. R. Greene, W. H. Kinney and G. Shiu, *Inflation as a probe of short distance physics*, Phys. Rev. D **64**, 103502 (2001) [arXiv:hep-th/0104102].
R. Easther, B. R. Greene, W. H. Kinney and G. Shiu, *Imprints of short distance physics on inflationary cosmology*, arXiv:hep-th/0110226.
R. Easther, B. R. Greene, W. H. Kinney and G. Shiu, *A generic estimate of trans-Planckian modifications to the primordial power spectrum in inflation*, Phys. Rev. D **66**, 023518 (2002) [arXiv:hep-th/0204129].

[4] G. Shiu and I. Wasserman, *On the signature of short distance scale in the cosmic microwave background*, Phys. Lett. B **536**, 1 (2002) [arXiv:hep-th/0203113].

[5] C. S. Chu, B. R. Greene and G. Shiu, *Remarks on inflation and noncommutative geometry*, Mod. Phys. Lett. A **16**, 2231 (2001) [arXiv:hep-th/0011241].

[6] U. H. Danielsson, *A note on inflation and transplanckian physics*, Phys. Rev. D **66**, 023511 (2002) [arXiv:hep-th/0203198].
U. H. Danielsson, *Inflation, holography and the choice of vacuum in de Sitter space*, JHEP **0207**, 040 (2002) [arXiv:hep-th/0205227].

[7] C. P. Burgess, J. M. Cline, F. Lemieux and R. Holman, *Are inflationary predictions sensitive to very high energy physics?*, JHEP **0302**, 048 (2003) [arXiv:hep-th/0210233].
C. P. Burgess, J. M. Cline and R. Holman, *Effective field theories and inflation*, arXiv:hep-th/0306079.
C. P. Burgess, J. M. Cline, F. Lemieux and R. Holman, *Decoupling, trans-Planckia and inflation*, arXiv:astro-ph/0306236.

[8] N. Kaloper, M. Kleban, A. E. Lawrence and S. Shenker, *Signatures of short distance physics in the cosmic microwave background*, Phys. Rev. D **66**, 123510 (2002) [arXiv:hep-th/0201158].
N. Kaloper, M. Kleban, A. Lawrence, S. Shenker and L. Susskind, *Initial conditions for inflation*, JHEP **0211**, 037 (2002) [arXiv:hep-th/0209231].

[9] A. Kempf, *Mode generating mechanism in inflation with cutoff*, Phys. Rev. D **63**, 083514 (2001)

[arXiv:astro-ph/0009209].
J. C. Niemeyer and R. Parentani, *Trans-Planckian dispersion and scale-invariance of inflationary perturbations*, Phys. Rev. D **64**, 101301 (2001) [arXiv:astro-ph/0101451].
A. Kempf and J. C. Niemeyer, *Perturbation spectrum in inflation with cutoff*, Phys. Rev. D **64**, 103501 (2001) [arXiv:astro-ph/0103225].
R. H. Brandenberger and J. Martin, *On signatures of short distance physics in the cosmic microwave background*, Int. J. Mod. Phys. A **17**, 3663 (2002) [arXiv:hep-th/0202142].
S. Shankaranarayanan, *Is there an imprint of Planck scale physics on inflationary cosmology?*, Class. Quant. Grav. **20**, 75 (2003) [arXiv:gr-qc/0203060].
J. C. Niemeyer, R. Parentani and D. Campo, *Minimal modifications of the primordial power spectrum from an adiabatic short distance cutoff*, Phys. Rev. D **66**, 083510 (2002) [arXiv:hep-th/0206149].
K. Goldstein and D. A. Lowe, *Initial state effects on the cosmic microwave background and trans-planckian physics*, Phys. Rev. D **67**, 063502 (2003) [arXiv:hep-th/0208167].
G. L. Alberghi, R. Casadio and A. Tronconi, *Trans-Planckian footprints in inflationary cosmology*, Phys. Lett. B **579**, 1 (2004) [arXiv:gr-qc/0303035].
J. Martin and R. Brandenberger, *On the dependence of the spectra of fluctuations in inflationary cosmology on trans-Planckian physics*, Phys. Rev. D **68**, 063513 (2003) [arXiv:hep-th/0305161].
N. Kaloper and M. Kaplinghat, *Primeval corrections to the CMB anisotropies*, arXiv:hep-th/0307016.
A. Ashoorioon, A. Kempf and R. B. Mann, *Minimum length cutoff in inflation and uniqueness of the action*, Phys. Rev. D **71**, 023503 (2005) [arXiv:astro-ph/0410139].

[10] L. Bergstrom and U. H. Danielsson, *Can MAP and Planck map Planck physics?*, JHEP **0212**, 038 (2002) [arXiv:hep-th/0211006].
O. Elgaroy and S. Hannestad, *Can Planck-scale physics be seen in the cosmic microwave background?*, Phys. Rev. D **68**, 123513 (2003) [arXiv:astro-ph/0307011].
T. Okamoto and E. A. Lim, *Constraining Cut-off Physics in the Cosmic Microwave Background*, Phys. Rev. D **69**, 083519 (2004) [arXiv:astro-ph/0312284].
J. Martin and C. Ringeval, *Superimposed Oscillations in the WMAP Data?*, Phys. Rev. D **69**, 083515 (2004) [arXiv:astro-ph/0310382].
J. Martin and C. Ringeval, *Addendum to "Superimposed Oscillations in the WMAP Data?"*, Phys. Rev. D **69**, 127303 (2004) [arXiv:astro-ph/0402609].
S. Hannestad and L. Mersini-Houghton, *A first glimpse of string theory in the sky?*, arXiv:hep-ph/0405218.
J. Martin and C. Ringeval, *Exploring the superimposed oscillations parameter space*, arXiv:hep-ph/0405249.
L. Sriramkumar and T. Padmanabhan, *Initial state of matter fields and trans-Planckian physics: Can CMB observations disentangle the two?*, arXiv:gr-qc/0408034.
R. Easther, W. H. Kinney and H. Peiris, *Observing trans-Planckian signatures in the cosmic microwave background*, arXiv:astro-ph/0412613.

[11] K. Schalm, G. Shiu and J. P. van der Schaar, *Decoupling in an expanding universe: Boundary RG-flow affects initial conditions for inflation*, arXiv:hep-th/0401164.

[12] M. Porrati, *Bounds on generic high-energy physics modifications to the primordial power spectrum from back-reaction on the metric*, arXiv:hep-th/0402038.

[13] J. Polchinski, *Renormalization and effective Lagrangians*, Nucl. Phys. B **231**, 269 (1984).

[14] V. Bozza, M. Giovannini and G. Veneziano, *Cosmological perturbations from a new-physics hypersurface*, JCAP **0305**, 001 (2003) [arXiv:hep-th/0302184].

[15] T. Tanaka, *A comment on trans-Planckian physics in inflationary universe*, arXiv:astro-ph/0012431.

[16] M. Lemoine, M. Lubo, J. Martin and J. P. Uzan, *The stress-energy tensor for trans-Planckian cosmology*, Phys. Rev. D **65** (2002) 023510 [arXiv:hep-th/0109128].

[17] M. Giovannini, *Assigning quantum mechanical initial conditions to cosmological perturbations*, Class. Quant. Grav. **20**, 5455 (2003) [arXiv:hep-th/0308066].

[18] R. H. Brandenberger and J. Martin, *Back-Reaction and the Trans-Planckian Problem of Inflation Revisited*, arXiv:hep-th/0410223.

[19] A. A. Starobinsky, *Robustness of the inflationary perturbation spectrum to trans-Planckian physics*, Pisma Zh. Eksp. Teor. Fiz. **73**, 415 (2001) [JETP Lett. **73**, 371 (2001)] [arXiv:astro-ph/0104043].
A. A. Starobinsky and I. I. Tkachev, *Trans-Planckian particle creation in cosmology and ultra-high energy cosmic rays*, JETP Lett. **76**, 235 (2002) [Pisma Zh. Eksp. Teor. Fiz. **76**, 291 (2002)] [arXiv:astro-ph/0207572].

[20] T. Banks and L. Mannelli, *De Sitter vacua, renormalization and locality*, Phys. Rev. D **67**, 065009 (2003) [arXiv:hep-th/0209113].
M. B. Einhorn and F. Larsen, *Interacting quantum field theory in de Sitter vacua*, Phys. Rev. D **67**, 024001 (2003) [arXiv:hep-th/0209159].
K. Goldstein and D. A. Lowe, *A note on alpha-vacua and interacting field theory in de Sitter space*, Nucl. Phys. B **669**, 325 (2003) [arXiv:hep-th/0302050].
M. B. Einhorn and F. Larsen, *Squeezed states in the de Sitter vacuum*, Phys. Rev. D **68**, 064002 (2003) [arXiv:hep-th/0305056].

[21] N. D. Birrell and P. C. W. Davies, *Quantum Fields In Curved Space*, Cambridge University Press, 1982.

[22] S. A. Fulling, *Aspects Of Quantum Field Theory In Curved Space-Time*, Cambridge University Press (London Mathematical Society Student Texts 17) 1989.

[23] F. Larsen, J. P. van der Schaar and R. G. Leigh, *de Sitter holography and the cosmic microwave background*, JHEP **0204**, 047 (2002) [arXiv:hep-th/0202127].
F. Larsen and R. McNees, *Inflation and de Sitter holography*, JHEP **0307**, 051 (2003) [arXiv:hep-th/0307026].
J. P. van der Schaar, *Inflationary perturbations from deformed CFT*, JHEP **0401**, 070 (2004) [arXiv:hep-th/0307271].
F. Larsen and R. McNees, *Holography, diffeomorphisms, and scaling violations in the CMB*, JHEP **0407**, 062 (2004) [arXiv:hep-th/0402050].

[24] H. V. Peiris *et al.*, *First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Implications for inflation*, Astrophys. J. Suppl. **148**, 213 (2003) [arXiv:astro-ph/0302225].

[25] P. Anderson, C. Molina-París, E. Mottola, arXiv:hep-th/0504134.

[26] E. Mottola, *Particle Creation In De Sitter Space*, Phys. Rev. D **31**, 754 (1985).
E. Mottola, *Thermodynamic Instability Of De Sitter Space*, Phys. Rev. D **33**, 1616 (1986).
E. Mottola, *A Quantum Fluctuation Dissipation Theorem For General Relativity*, Phys. Rev. D **33**, 2136 (1986).
P. Mazur and E. Mottola, *Spontaneous Breaking Of De Sitter Symmetry By Radiative Effects*, Nucl. Phys. B **278**, 694 (1986).
J. H. Traschen and C. T. Hill, *Instability Of De Sitter Space On Short Time Scales*, Phys. Rev. D **33**, 3519 (1986).
I. Antoniadis, J. Iliopoulos and T. N. Tomaras, *Quantum Instability Of De Sitter Space*, Phys. Rev. Lett. **56**, 1319 (1986).

[27] U. H. Danielsson, *Transplanckian energy production and slow roll inflation*, arXiv:hep-th/0411172.

[28] M. Porrati, *Effective field theory approach to cosmological initial conditions: Self-consistency bounds and non-Gaussianities*, arXiv:hep-th/0409210.

[29] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister and S. P. Trivedi, *Towards inflation in string theory*, JCAP **0310**, 013 (2003) [arXiv:hep-th/0308055].
S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, *De Sitter vacua in string theory*, Phys. Rev. D **68** (2003) 046005 [arXiv:hep-th/0301240].