

**BLACK HOLE CONDENSATION AND
THE UNIFICATION OF STRING VACUA**

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It is argued that black hole condensation can occur at conifold singularities in the moduli space of type II Calabi–Yau string vacua. The condensate signals a smooth transition to a new Calabi–Yau space with different Euler characteristic and Hodge numbers. In this manner string theory unifies the moduli spaces of many or possibly all Calabi–Yau vacua. Elementary string states and black holes are smoothly interchanged under the transitions, and therefore cannot be invariantly distinguished. Furthermore, the transitions establish the existence of mirror symmetry for many or possibly all Calabi–Yau manifolds.

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1. Introduction and summary

In ten dimensions, there is a small number of consistent string theories. When first discovered, this near-uniqueness raised the hope that, despite the inherent difficulties in extrapolating from the Planck scale to the weak scale, it might be possible to obtain testable predictions from string theory. This hope was greatly diminished by the discovery of a plethora of four-dimensional string vacua. In order to compute the low-lying spectrum and couplings it is apparently first necessary to choose among many thousands of Calabi–Yau spaces (or more general conformal field theories). The process of compactification appeared to ruin the uniqueness of ten-dimensional string theory, along with the predictive power it entailed.

In this paper we will argue that the situation is in fact much better than it appears. In the context of type II strings, we will see that many, and possibly all, of these Calabi–Yau vacua are in fact different branches of a vastly larger “universal” moduli space.¹ The branches are connected in a smooth and calculable manner by black hole condensation which can occur at “conifold” points of the moduli space. This condensation can not be described in the language of conformal field theory, and so is not constrained to preserve quantities such as the number of light generations which are topological invariants of conformal field theory. Thus the number of distinct four-dimensional string vacua is much smaller than previously suspected. Indeed, it is conceivable that there is a unique four-dimensional string vacuum.

Of course even if the goal of tying together all type II string vacua is realized, we are still quite far from making testable predictions. One must extend these ideas to theories with $N = 1$ supersymmetry in four dimensions, and then understand how a superpotential is generated which lifts the continuous vacuum degeneracy and breaks $N = 1$ down to $N = 0$. Even then there is no guarantee that there is a unique or small number of vacua. Nevertheless we feel the time may be ripe for progress on all these problems.

It has long been known in the mathematics literature [3–6] that it is possible to travel from one Calabi–Yau to another by degenerating certain three-cycles and then blowing them back up as two-cycles, thereby changing the Hodge numbers. This process enables one to glue together Calabi–Yau moduli spaces along the subspaces corresponding to conifolds.

¹ A previous step in this direction was taken in [1,2], where string compactifications which are topologically distinct as Calabi–Yau spaces but not as conformal field theories were smoothly connected.

Indeed it has been conjectured [7] that all Calabi–Yau spaces are connected in this fashion. The relevance of these results to string theory was advocated in a series of prescient papers [8–11]. However the construction, at the level of both mathematics and conformal field theory, is singular, and no proposal was made for how string theory might physically implement the transition from one Calabi–Yau to its neighbor. The purpose of the present work is to describe such a mechanism.

The key to understanding this transition is in a recent resolution [12] of simple conifold singularities in type II string theory. Near a conifold, the moduli space metric becomes singular. At the same time, string theory contains black hole hypermultiplets which are degenerating to zero mass. It was shown [12] that the Wilsonian effective field theory including the light black holes is smooth near the conifold, and that integrating out the light black holes reproduces the singularity.

The singularities of the conifolds which glue together Calabi–Yau moduli spaces are more complicated than the simple type analyzed in [12]. In this paper we shall again find that these singularities are resolved by light black hole hypermultiplets, but there are in general many such hypermultiplets. The potential V for these hypermultiplets is determined by $N = 2$ supersymmetry. V has flat directions, along which the black holes can condense and give masses to vector multiplets. In this way one discovers a new branch of the moduli space with different numbers of hypermultiplets and vector multiplets. In a IIB string theory the number of massless vector multiplets (hypermultiplets) is h_{21} ($h_{11}+1$) and this new branch corresponds to a topologically distinct Calabi–Yau space. In this paper we analyze only the transition from the quintic in \mathbb{P}^4 with Hodge numbers $(h_{21}, h_{11}) = (101, 1)$, to a variety in $\mathbb{P}^4 \times \mathbb{P}^1$, with Hodge numbers $(86, 2)$, but our construction clearly generalizes. Further analysis will appear in a forthcoming publication [13].

Our construction has a number of implications that are worth emphasizing at the outset. First, as mentioned, type II string vacua which were previously thought to be disjoint are now seen to fit together into a connected web with string physics smoothly interpolating from one component to another. Second, as we move along such an interpolating path, the spacetime background of our string theory undergoes a drastic change in topology. Unlike the spacetime topology change of [1,2], in which Hodge numbers stay fixed while more subtle topological invariants (such as the intersection form) change, here we find that string theory is perfectly smooth even as the Hodge numbers jump. Third, using these results we can vastly extend the mirror symmetry construction of [14]. Namely, on general grounds, once one proves the existence of mirror symmetry for one point in a given moduli

space, via deformation arguments one can conclude the existence of mirror symmetry at all other points lying in the same connected component. This was used in [14], for instance, to argue for mirror symmetry throughout a Calabi–Yau moduli space so long as it contains a minimal model point at which an explicit mirror partner exists. Now we see that our deformation arguments are not limited to a single Calabi–Yau moduli space, but rather extend to all Calabi–Yau manifolds connected by conifold transitions; the latter includes essentially all known Calabi–Yau manifolds.² Finally, our construction provides food for thought on the fascinating interplay between strings and black holes. A degenerating black hole hypermultiplet is reinterpreted as a fundamental string excitation (corresponding to a modulus) after crossing the transition. Thus there can be no fundamental distinction between strings and black holes: they smoothly transform into one another.

2. Classical structure of conifolds

For concreteness, we focus on a particular example in this paper, although our analysis is general. The moduli space \mathcal{M} of all complex structures on a quintic threefold X can be described in terms of the defining equations of the quintics: the general such equation takes the form of a homogeneous quintic polynomial

$$c_0 x_0^5 + \cdots + c_{125} x_4^5 = 0 \tag{2.1}$$

and involves 126 coefficients. Some 25 of these are redundant due to the action of $\mathrm{GL}(5)$ on the homogeneous coordinates $[x_0, \dots, x_4]$, leaving 101 independent parameters in the moduli space.

For general values of (c_0, \dots, c_{125}) , the equation (2.1) defines a nonsingular Calabi–Yau manifold X_c in \mathbb{P}^4 , but at certain special values (along a set Δ of complex codimension 1 in the parameter space), the solution set of (2.1) becomes singular. These singularities were analyzed many years ago by Lefschetz [15], who showed that:

² In [9,10] it was established that all simply-connected Calabi–Yau manifolds *known at that time* could be connected in this way. (It is easiest to deal with multiply connected Calabi–Yau’s by working with their covering spaces.) Since the time of [9,10] a number of new constructions of Calabi–Yau manifolds have been proposed, and while it is clear that most of these can be connected, no systematic study has been made of this question.

1) the “generic” singular space X_c , $c \in \Delta$ has a single node,³ i.e., a singular point with local equation $\sum_{i=1}^4 y_i^2 = 0$,

2) the singular point determines a “vanishing cycle” $\gamma \in H_3(X, \mathbb{Z})$ which shrinks to zero size when the singularity is approached,⁴ and

3) the homology of the Calabi–Yau manifolds undergoes a monodromy transformation

$$\delta \mapsto \delta + \langle \delta | \gamma \rangle \gamma \quad (2.2)$$

upon transport around a loop in \mathcal{M} encircling the singular locus Δ . (Here, $\langle \delta | \gamma \rangle$ denotes the number of (oriented) intersections of δ with γ .) If all of the singular points on X_c are nodes, then X_c is called a “Calabi–Yau conifold” (so named [9] because of the conical nature of the singularities).

If we introduce a holomorphic 3-form Ω (depending on the moduli), and use the “periods” $\int_\delta \Omega$ of Ω to describe the complex structure, we find that some of the periods become multiple-valued near the singular locus. In fact, if we let

$$Z \equiv \int_\gamma \Omega \quad (2.3)$$

be the period corresponding to the vanishing cycle, then we find that the singular locus Δ in \mathcal{M} is locally described by $Z = 0$, and that other periods must take the form

$$\int_\delta \Omega \sim \frac{1}{2\pi i} \langle \delta | \gamma \rangle Z \ln Z + (\text{single-valued function}) \quad (2.4)$$

near $Z = 0$ in order to have the correct monodromy property.

Let us now consider the set of quintic conifolds in \mathbb{P}^4 which have k singular points. Since asking for a single node places a single condition on the parameters, one’s initial expectation is that asking for a conifold with k singular points will place k conditions on the parameters, leading to a locus of complex codimension k in \mathcal{M} . When this is true, the generic point of that locus will locally be an intersection of k hypersurfaces, meeting transversally. Near the intersection of all of these, there will be k different monodromy transformations with vanishing cycles $\gamma^1, \dots, \gamma^k$, and the periods must take the form

$$\int_\delta \Omega \sim \frac{1}{2\pi i} \sum_{a=1}^k \langle \delta | \gamma^a \rangle Z^a \ln Z^a + (\text{single-valued function}) \quad (2.5)$$

³ We distinguish between the singular points of the Calabi–Yau—here called *nodes*—and the points in the moduli space which label such Calabi–Yau’s—called *conifold points*.

⁴ More precisely, the period integrals over γ vanish in the limit.

near $Z^1 = \dots = Z^k = 0$, where $Z^a \equiv \int_{\gamma^a} \Omega$ are among the local coordinates near the intersection.

However, in the example which we will study in detail there are $k = 16$ singular points on the conifold which impose only 15 conditions on the parameters. In fact, the vanishing cycles in our example in the next section will satisfy the homology relation

$$\sum_{a=1}^{16} \gamma^a = 0, \quad (2.6)$$

which implies a corresponding relation among the periods:

$$\sum_{a=1}^{16} Z^a = 0. \quad (2.7)$$

The locus of conifolds with (at least) 15 singular points is described by $Z^1 = \dots = Z^{15} = 0$, but since the hypersurface $Z^{16} = 0$ passes through this locus (as a consequence of (2.7)), the 16th point is also present, without imposing any further conditions. This is clearly a special property of the particular collection of 16 points which we are considering.

In this situation, we have 16 monodromy transformations near a locus of codimension 15. The asymptotic form (2.5) still holds near $\{Z^a = 0\}$, but the γ^a and Z^a are related by (2.6) and (2.7). In particular, only a subset of the Z^a 's can be included among a list of local coordinates near the intersection locus.

3. The example

The example we study first appeared in the physics literature in [10]. We will describe this example precisely in what follows, but first let us emphasize the basic idea. We start with a smooth quintic in \mathbb{P}^4 and follow a path in its complex structure moduli space leading us to a conifold with 16 singular points. Each of these singular points can be described locally as a cone over an $S^2 \times S^3$ base. We resolve such singularities by cutting out a neighborhood of the singular point and gluing in a smooth space (of real dimension 6) whose boundary agrees with that of the extracted set, namely $S^2 \times S^3$. For the singular points of a conifold this can be done in two ways: *i*) glue in $B^3 \times S^3$ or *ii*) glue in $S^2 \times B^4$. The former is a deformation of the conifold back into the quintic moduli space, by giving positive volume to the shrunken three-cycles. The latter is a small resolution of the conifold by giving positive area to the S^2 's. Effectively, the small resolution replaces previous S^3 's

with S^2 's and thereby changes the Hodge numbers, and hence topology, of the Calabi–Yau space. (In fact, there are generally two distinct ways of performing the small resolution that differ by a flop. This played a key role in [1,2] but is not of central importance here.) Our goal is to understand how type II string theory behaves as we attempt to pass from the smooth quintic, through the conifold, and on to its topologically distinct small resolution. To do so, we first recast the present discussion into a more concrete form.

Consider the set of quintics in \mathbb{P}^4 which contain a fixed \mathbb{P}^2 , say the one with $x_3 = x_4 = 0$. We do this because, as we shall see, the 16 singular points referred to above can be made to all reside on this \mathbb{P}^2 . The defining equation of such a Calabi–Yau space must not contain any of the 21 monomials x_0^5, \dots, x_2^5 which involve only x_0, x_1 and x_2 ; there are therefore 105 parameters in the defining equation. On the other hand, the number of redundancies has decreased, since we are only free to use the subgroup of $GL(5)$ which fixes the \mathbb{P}^2 , i.e., matrices (a_{jk}) for which the coefficients a_{jk} vanish when $j = 3, 4$ and $k = 0, 1, 2$. That subgroup has dimension 19, so the total number of effective parameters is 86, a set of codimension 15 in the original 101-dimensional space \mathcal{M} .

If we write the defining polynomial of such a quintic in the form

$$f(x) = x_3 g(x) + x_4 h(x) \tag{3.1}$$

where $g(x)$ and $h(x)$ are polynomials of degree 4, then by considering the partial derivatives of f with respect to x_3 and x_4 it becomes apparent that the quintic must be singular along the set $\{x_3 = x_4 = g(x) = h(x) = 0\}$ which consists of the sixteen points of intersection of g and h within \mathbb{P}^2 . When g and h are generic, these singularities are simply nodes. That is, we have defined a Calabi–Yau *conifold* rather than a Calabi–Yau *manifold*.

The vanishing cycles of these singular points are easy to locate. To do so, consider a neighborhood of a given singular point on the conifold that we obtain by intersecting the quintic with a ball in \mathbb{P}^4 , i.e., a B^8 . As discussed in [10,11], this intersection is a cone with base being $S^2 \times S^3$. Therefore, if we remove 16 such small balls from \mathbb{P}^4 about the 16 singular points, then at each point we remove a singular portion from the Calabi–Yau whose boundary is topologically $S^2 \times S^3$. It is then possible to glue in 16 copies of $B^3 \times S^3$ to obtain the smooth quintic Calabi–Yau manifold. Note that the singular Calabi–Yau contains \mathbb{P}^2 as a smooth 4-manifold passing through the 16 singular points. When we remove the 16 balls, we remove 16 B^4 's from this \mathbb{P}^2 . The boundary of each such B^4 is the S^3 we have glued in to desingularize the space, i.e., the vanishing cycles. The \mathbb{P}^2 with

16 B^4 's removed is thus a 4-manifold-with-boundary on our Calabi–Yau manifold, and its boundary is precisely the sum of all the vanishing cycles γ^a . That is, (2.6) holds in the homology group. Some relation such as (2.6) was to be expected from our count of the codimension.

The singular quintics can alternatively be given a small resolution by blowing up the \mathbb{P}^2 contained within them. We blow up the locus in \mathbb{P}^4 defined by $x_3 = x_4 = 0$, which can be modeled inside $\mathbb{P}^4 \times \mathbb{P}^1$ as the set where $\{y_0 x_4 - y_1 x_3 = 0\}$, $[y_0, y_1]$ being homogeneous coordinates on \mathbb{P}^1 . The original Calabi–Yau conifold is blown up to a Calabi–Yau manifold defined by

$$\begin{aligned} y_0 x_4 - y_1 x_3 &= 0 \\ y_0 g(x) + y_1 h(x) &= 0. \end{aligned} \tag{3.2}$$

Topologically, this small resolution process glues in a copy of $S^2 \times B^4$ along each boundary $S^2 \times S^3$. There is a new $(1, 1)$ class on the resolved space, which measures the area of the new S^2 's which were added. (All have the same area.)

4. Quantum structure of conifolds

In this section we will show that there is a smooth Wilsonian effective theory at the conifold which includes light black holes, and that one can recover the classical structure of conifolds (as described in section 2) by integrating out these light black holes. The argument follows [12], where further discussion can be found.

To make contact with the usual formulation of $N = 2$, $d = 4$ supergravity and special geometry [16,17], we introduce a symplectic basis of three-cycles on the quintic

$$\langle A_I | B^J \rangle = -\langle B^J | A_I \rangle = \delta_I^J, \quad \langle A_I | A_J \rangle = \langle B^I | B^J \rangle = 0, \tag{4.1}$$

and corresponding periods

$$\begin{aligned} F_I &= \int_{A_I} \Omega, \\ Z^J &= \int_{B^J} \Omega. \end{aligned} \tag{4.2}$$

Only $30 = 15 + 15$ of the 204 periods are relevant for our purposes, so we henceforth restrict

$$I, J = 1, \dots, 15. \tag{4.3}$$

The Z^K provide good local coordinates on \mathcal{M} , and γ^a may be expanded

$$\gamma^a = n^a{}_I B^I. \quad (4.4)$$

To be specific we choose a basis such that

$$\begin{aligned} n^{16}{}_I &= -1, \\ n^a{}_I &= \delta^a{}_I, \quad a = 1, \dots, 15. \end{aligned} \quad (4.5)$$

For a compactification of type IIB string theory on X , the four dimensional field theory has 101 $N = 2$ vector multiplets with associated $U(1)$ field strengths which descend from the self-dual five form F in ten dimensions. We are interested in the 15 four-dimensional $U(1)$ field strengths G_I descended from F as

$$F = (1 + *)G_I \alpha^I, \quad (4.6)$$

where $*$ is the ten-dimensional Hodge dual and α^I is a harmonic three-form dual to A_I .

The ten-dimensional type IIB theory contains extremal black threebranes [18] which can wrap around any one of the 16 degenerating cycles γ^a and appear as an extremal black hole in four dimensions. The mass m^a of the a th such black hole is determined from the Bogolmony bound [19] up to a constant as

$$m^a = |Z^a| = |n^a{}_I Z^I|, \quad (4.7)$$

while the charge associated to the I th $U(1)$ is

$$q^a{}_I = n^a{}_I. \quad (4.8)$$

Evidently there are 16 charged hypermultiplets which can become light as we approach a conifold point in the quintic moduli space. We shall denote these by H^a . As in [20,12], there is a smooth Wilsonian theory near the conifold which includes the light fields H^a . Consider a generic region near a codimension one conifold locus $Z^K = 0$ for which only one three-cycle B^K vanishes. It is evident from (4.7) that only one field H_K becomes light. Integrating out H_K leads to effective couplings between the vector multiplets which run (as a function of the moduli) according to their one-loop beta functions. These running couplings are characterized by the holomorphic section [16]

$$\tau_{IJ} \sim \frac{1}{2\pi i} \delta_{IJ} \ln Z^J + (\text{single-valued function}). \quad (4.9)$$

Using the special geometry relation

$$\tau_{IJ} = \partial_I F_J, \quad (4.10)$$

and integrating one finds the one-loop correction to the effective periods near the hypersurface $Z^K = 0$:

$$F_K \sim \frac{1}{2\pi i} Z^K \ln Z^K + (\text{single-valued function}), \quad (4.11)$$

exactly as in [12]. This agrees with the classical result (2.5).

At the conifold locus where γ^{16} degenerates, $\sum_K Z^K = 0$ and the field H^{16} becomes light. In the basis (4.5), this field carries charge minus one with respect to each of the 15 U(1)s. Consequently there will be an identical contribution to the beta function of all couplings, and τ will behave as

$$\tau_{IJ} \sim \frac{1}{2\pi i} \ln \sum_{K=1}^{15} Z^K + (\text{single-valued function}). \quad (4.12)$$

This implies a monodromy for each of the 15 periods F_L about this codimension one locus in the moduli space:

$$F_L \sim \frac{1}{2\pi i} \sum_{J=1}^{15} Z^J \ln \sum_{K=1}^{15} Z^K + (\text{single-valued function}) \quad (4.13)$$

Comparing with (2.5), we see that the monodromies computed by integrating out the light fields of the Wilsonian effective field theory are in complete agreement with those of the classical computation. The singularities of the classical moduli space metric follow from (4.11), (4.13) and relations of special geometry. Hence they are also reproduced from the smooth Wilsonian theory.

It was crucial in our analysis that we regarded each of the 16 states obtained by wrapping threebranes around degenerating three-cycles as quanta of independent quantum fields. This is motivated by consideration of the large radius limit of the quintic conifold. In this limit the singular points are widely separated, and the black holes correspond to well-localized threebranes wrapping around degenerating three-cycles. Because they are well-separated it is natural to quantize them as independent objects. This critical assumption is an adaptation of the one made in [12], justified in part by the overall consistency of the picture presented here and in the next section. As pointed out in [12], it would be desirable to find an independent assessment of the validity of this method of counting states.

5. Black hole condensation

So far the story is similar in spirit, although more involved in detail, to the case of the simple conifold with a single vanishing cycle considered in [12]. However inspection of the effective theory near the conifold reveals a dramatic new feature. Each hypermultiplet H^a contains two charged complex scalars which we denote $h^{a\alpha}$ where $\alpha = 1, 2$ is the global $SU(2)_R$ index of our $N = 2$ representation. This gives a total of 32 complex scalar fields. Supersymmetry implies a potential for these scalar fields of the general form [16,21] (taking the vevs of the scalar components of the vector multiplets to vanish)

$$V \sim \sum_{I,J=1}^{15} M^{IJ} D_I^{\alpha\beta} D_{\alpha\beta J} \quad (5.1)$$

where M^{IJ} is a positive definite matrix [16,21] and

$$D_I^{\alpha\beta} = \sum_{a=1}^{16} q_I^a (h^{*a\alpha} h^{a\beta} + h^{*a\beta} h^{a\alpha}). \quad (5.2)$$

This potential has a flat direction along which the three independent components of $D_I^{\alpha\beta}$ vanish,

$$D_I^{\alpha\beta} = 0. \quad (5.3)$$

This gives 45 real constraints on the 32 complex fields. In addition there are 15 gauge transformations which rotate the fields, leaving 4 real vacuum parameters. Up to a gauge transformation the general solution of (5.3) in the basis (4.5) is

$$h^{a\alpha} = v^\alpha \quad \text{for all } a \quad (5.4)$$

for any complex two vector v . Moving along the flat direction, the black holes condense and we see that their moduli space is parametrized by a single hypermultiplet. The conifold point in the space of quintics (at which all 16 cycles vanish) corresponds to $v = 0$. Moving away from this point along the flat direction corresponds to giving a vev to the charged hypermultiplets. This vev breaks all 15 $U(1)$'s. Thus we have discovered a second branch of the moduli space corresponding to a charged black hole condensate. This branch has $101 - 15 = 86$ massless vector multiplets, and $2 + 1 = 3$ massless hypermultiplets.

Compactification of IIB string theory on a Calabi–Yau with $(h_{21}, h_{11}) = (86, 2)$ leads to 86 vector multiplets and 3 hypermultiplets. A space with precisely these Hodge numbers

arises if the singular conifold with 16 degenerate cycles is resolved with a \mathbb{P}^1 , as discussed in section 3. It is natural to identify the new branch of the moduli space discovered in the analysis of the Wilsonian effective field theory at the conifold with compactification on the $(86, 2)$ Calabi–Yau. Further evidence for this identification can be obtained by analyzing the behavior of the theory from the $(86, 2)$ side and will be presented in [13].

While analysis of the general case will be deferred to later work, one salient feature is worth mentioning. A general conifold has P vanishing cycles which obey Q homology relations of the type (2.6). This implies P degenerating black hole hypermultiplets which carry charge with respect to $P - Q$ $U(1)$ s. The generalization of the potential (5.1) will then have Q flat directions, since there are $P - Q$ equations of the form (5.3) for P hypermultiplets. Generically all $P - Q$ $U(1)$ s will be broken along these flat directions. Hence the new branch of the moduli space will have $P - Q$ fewer vector multiplets and Q more hypermultiplets, corresponding to a Calabi–Yau space with Hodge numbers $(h_{21} - P + Q, h_{11} + Q)$. This counting agrees precisely with one made using the algebraic geometry methods of [3, 22, 23] to compute the Hodge numbers of a Calabi–Yau space obtained by degenerating and blowing up cycles.

6. Mirror symmetry

Two Calabi–Yau manifolds are said to constitute a mirror pair if their corresponding conformally invariant nonlinear sigma models are isomorphic via a mapping that flips the sign of an eigenvalue of a $U(1)$ symmetry contained in the $N = 2$ superconformal algebra. Such conformal theories can be used as the internal part of a string model, and, as we have discussed, accurately describe string physics so long as we are sufficiently far away from conifold points. We have learned in the above discussion that even though the conformal field theory description of the string model breaks down at conifold points in the moduli space, the string description is perfectly well behaved. Thus, as in [24], it seems worthwhile to emphasize the notion of *string equivalence*: two geometric spaces are said to be string equivalent if they give rise to isomorphic models when taken as the background spacetime for string theory. Mirror symmetry is therefore a special case of string equivalence in which the explicit isomorphism takes the form noted above. More precisely, this isomorphism leads one to define, in the context of type II string theory, two Calabi–Yau spaces as constituting a mirror pair if the type IIA string on the first is isomorphic to the type IIB

string on the second. Away from conifold points, where conformal field theory is valid, this is essentially equivalent⁵ to the standard formulation of mirror symmetry.

Recall that in [14] a construction of pairs of mirror manifolds was presented which relied crucially on the existence of special points in moduli space at which the associated Calabi–Yau has enhanced discrete symmetries. Nonetheless, this construction was shown to establish the existence of mirror pairs away from such special points through deformation arguments. Namely, if M and W constitute a mirror pair at one point in the moduli space, then we can generate a family of mirrors by deforming, say, the complex structure of M and, correspondingly, the Kähler structure of W (and vice versa). The details of this notion were made precise in [1], but the essential idea is simple: since M and W give isomorphic physics, whatever operation is performed on M has a physically isomorphic description as an operation on W , thereby maintaining the mirror relationship.

This argument requires that the operation, say a deformation of the theory, be smooth; otherwise we lose control and have no basis for drawing any conclusions. For this reason, such deformation arguments have only been used to establish mirror symmetry for a continuously connected (in the sense of conformal field theory) family of Calabi–Yau spaces containing at least one point at which the explicit construction of [14] could be applied. We now learn, from the results of the present work, that we can continue such deformation arguments through conifold transitions since string theory is perfectly well behaved along such a path. Thus, given *one point* in the web of connected Calabi–Yau spaces at which we can establish the existence of a mirror partner (the construction of [14] gives us many such points) we can use our deformation arguments to establish the existence of mirror symmetry at *all* points in the web. As almost all known Calabi–Yau manifolds are connected to the web this establishes mirror symmetry for most, or possibly all, Calabi–Yau spaces.⁶

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⁵ Some care must be taken concerning the integral structures [25].

⁶ The idea of using conifold transitions to yield a more general mirror construction was proposed some time ago [26]; the present work is the physical realization of that mathematical conjecture.

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